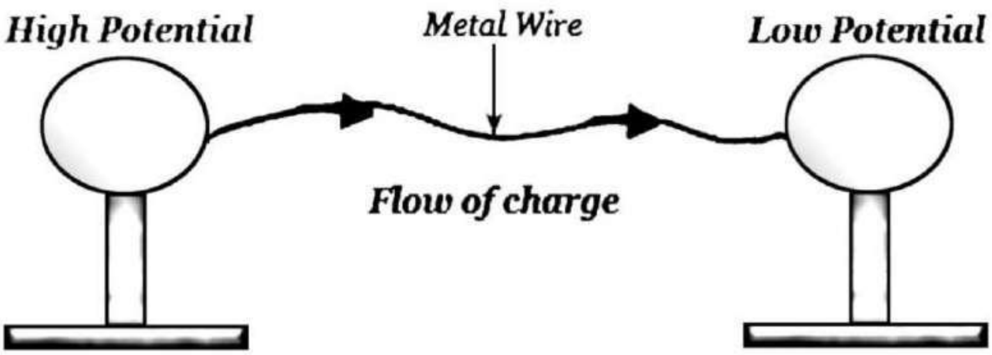
UNIT- 2

Electric Potential & Capacitance

Introduction

The electrostatic potential of a charged body represents the degree of electrification of the body. It determines the direction of flow of charge between two charged bodies placed in contact with each other.

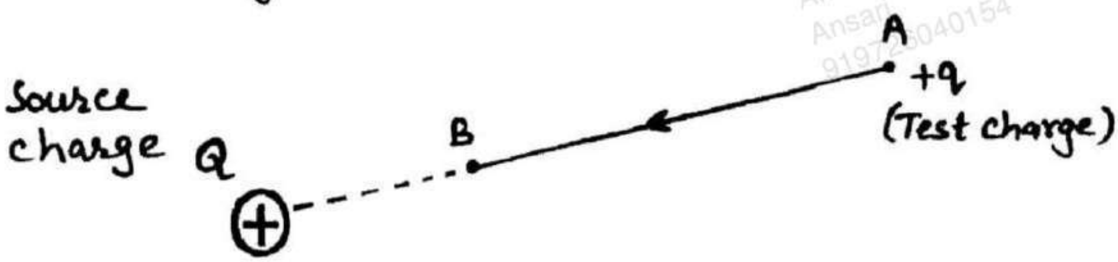
The charge always flows from a body at higher potential to another body at lower potential. The flow of charge stops as soon as the potentials of the bodies become equal.



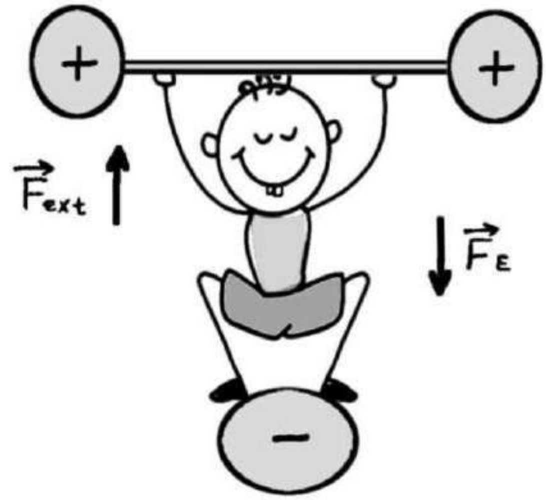
Electrostatic Potential Energy

Let us assume an electric field $\vec{E}$ due to a charge $+Q$ placed at the origin.

Let a small test charge $+q$ be brought from a point A to a point B against the repulsive force on it due to charge $+Q$.



We assume that the external force $\vec{F}_{ext}$ applied is just sufficient to counter the repulsive electric force $\vec{F}_E$ on the test charge q so the net force on test charge q is zero and it moves from A to B without any acceleration.



Now the work done by external force is negative of work done by the electric force and gets fully stored in the charge q in the form of its potential energy.

Work done by external force in moving the test charge +q from A to B is -

$$\boxed{W_{AB} = \int_A^B \vec{F}_{ext} \cdot \vec{dr} = -\int_A^B \vec{F}_E \cdot \vec{dr}}$$

This work done against electrostatic force gets stored as potential energy.

$$\boxed{\Delta U = U_B - U_A = W_{AB}}$$

Hence electrostatic potential energy difference between two points B and A is equal to the minimum work done by an external force in moving a test charge q from A to B without acceleration.

Note :-

The work done by an electrostatic field in moving a given charge from one point to another depends only on the positions of initial and final points. It does not depend on the path chosen in going from one point to the other.

* For a charge distribution of finite extent, we choose zero electrostatic potential energy at infinity. [$U_\infty = 0$]

Therefore, when point A is at infinity we get.

$$W_{\infty B} = U_B - U_\infty$$

Or $$W_{\infty B} = U_B - 0 \qquad \boxed{W_{\infty B} = U_B}$$

Hence the electrostatic potential energy of a charge q at a point in an electric field due to any charge configuration is equal to the work done by the external force in bringing the charge q from infinity to that point without any acceleration.

Electrostatic Potential

The electric potential difference between two points B and A in an electric field is equal to the amount of work done in carrying unit positive test charge from A to B without acceleration along any path between the two points.

If V_A and V_B are the electrostatic potentials at A and B respectively then

$$\boxed{\frac{U_B - U_A}{q} = \frac{W_{AB}}{q} = V_B - V_A}$$

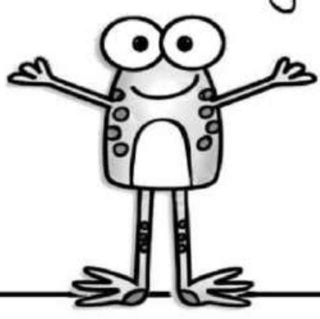
We can define electric potential at any point by choosing the potential to be zero at infinity.
So if we take point A at infinity then.

$$\boxed{V_B = \frac{W_{\infty B}}{q}}$$

* The electrostatic potential is a scalar quantity. Its SI unit is Volt.

1 Volt :-

The electrostatic potential difference between any two points in an electrostatic field is said to be one volt when 1 Jule of work is done in moving a positive charge of one coulomb from one point to the other against the electrostatic force of the field.



$$1\,V = \frac{1\,J}{1\,C}$$

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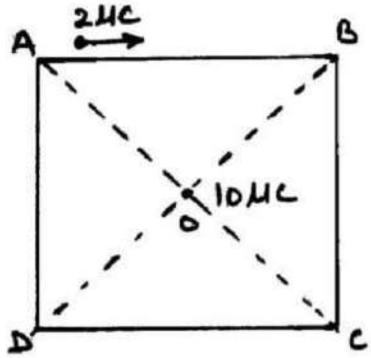
$$= 1\,JC^{-1} = 1\,NmC^{-1}$$

Q. What is work done in moving a $2\mu C$ point charge from corner A to corner B of a square ABCD when a $10\mu C$ charge exist at the centre of the square.



Sol. As here $\boxed{V_A = V_B}$

So the work done -

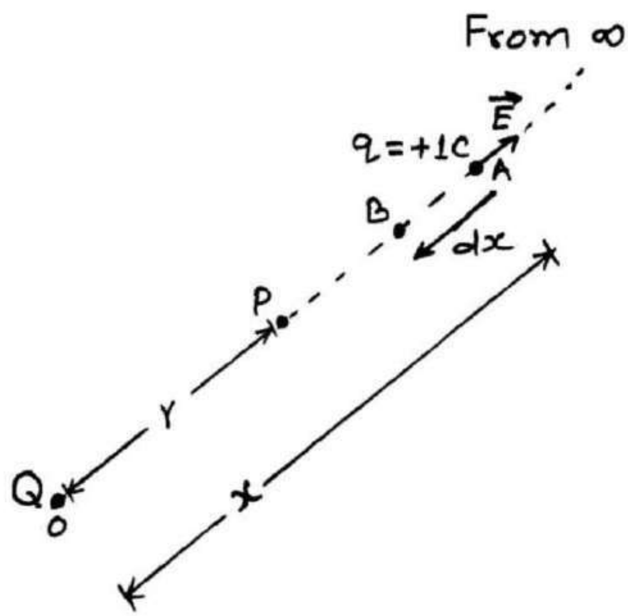
$$W = q(V_B - V_A)$$

$$W = \text{zero}$$

Electrostatic Potential due to a Point Charge

Suppose we have to calculate electrostatic Potential at any point P due to a Single point charge +Q at O, where OP = r.

By definition electrostatic potential at P is the amount of work done in carrying a unit positive charge from ∞ to P.



At some intermediate point A on this path, where OA = x, the electrostatic force on unit positive charge is -

$$F = QE = E = \frac{Q}{4\pi\epsilon_0 x^2} \quad (\text{along } OA)$$

Now small amount of work done in moving a unit positive charge from A to B where $\vec{AB} = \vec{dx}$ is

$$dW = \vec{E}\cdot\vec{dx} = E\,dx\cos 180^\circ = -E\,dx$$

$\therefore$ Total work done in moving positive unit charge from ∞ to the point P is -

$$W = \int_{\infty}^{r} -E\,dx = \int_{\infty}^{r} -\frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}\,dx$$

$$W = -\frac{Q}{4\pi\epsilon_0}\int_{\infty}^{r} x^{-2}\,dx = -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{r}$$

So $$W = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{Q}{4\pi\epsilon_0 r}$$

So by definition the electrostatic potential at P due to charge +Q is -

$$V = W \qquad \boxed{V = \frac{Q}{4\pi\epsilon_0 r}}$$

* When Q is positive, the electrostatic potential V is positive and when Q is negative, electrostatic potential is negative.

* Electrostatic potential due to a single charge is spherically symmetric.

$$\boxed{V \propto \frac{1}{r}}$$

$$\boxed{E \propto \frac{1}{r^2}}$$

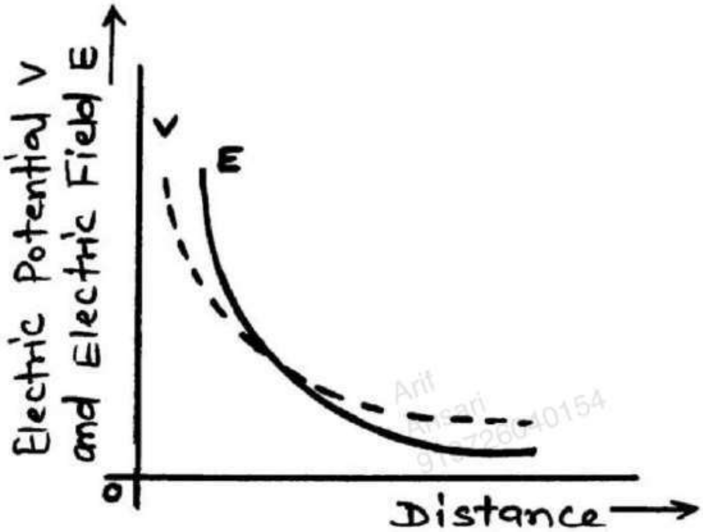
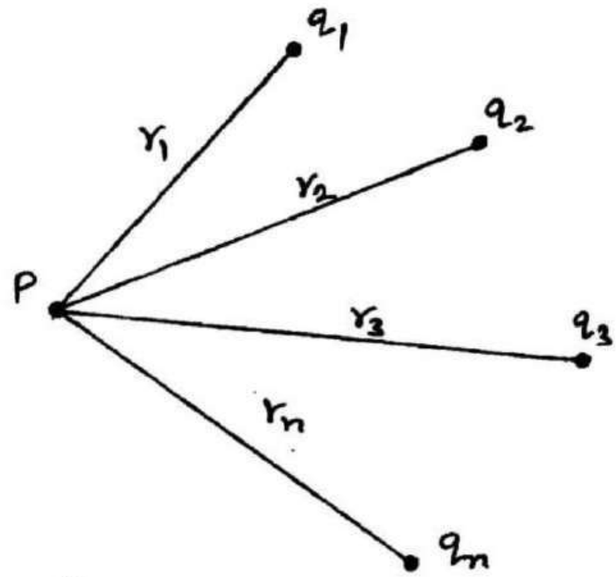


Fig:- Variation of Electrostatic Potential and Electric field with distance.

Potential at a Point due to group of Electric Charges :-

Suppose there are a number of point charges $q_1, q_2, q_3, \ldots q_n$ at distances $r_1, r_2, r_3, \ldots r_n$ respectively from the point P, where electric potential is to be calculated.



The potential at P due to various charges are -

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1}, \quad V_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2}$$

$$V_3 = \frac{1}{4\pi\epsilon_0}\frac{q_3}{r_3} \quad \text{and so on} \quad V_n = \frac{1}{4\pi\epsilon_0}\frac{q_n}{r_n}$$

The resultant potential at P is

$$V = V_1 + V_2 + V_3 + \ldots V_n$$

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots \frac{q_n}{r_n}\right)$$

or

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}}$$

Q. What is the electrostatic potential at the surface of a gold nucleus of diameter 13.2 fermi. Z for gold is 79.

Sol. Here $r = \frac{13.2}{2} = 6.6$ fermi

or $r = 6.6 \times 10^{-15}$ m. and $q = ne$

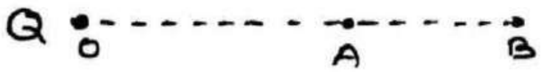
so $q = 79 \times 1.6 \times 10^{-19}$

So $V = \frac{q}{4\pi\epsilon_0 r} = \frac{9 \times 10^{9} \times 79 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-15}}$

$V = 1.7 \times 10^{7}$ Volt.

Q. (CBSE 2016) A point charge Q is placed at point O as shown in the fig. The potential difference $V_A - V_B$ is positive. Is the charge Q negative or positive?

Sol.



We know that, $V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$

Here $V \propto \frac{1}{r}$

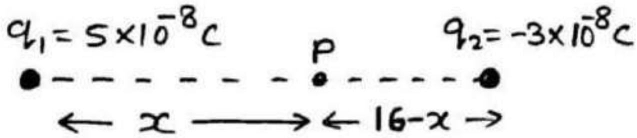
The potential due to a point charge decreases with increase in distance.

$V_A - V_B > 0 \Rightarrow V_A > V_B$

Hence the charge Q is positive.

Q. Two charges 5×10^{-8}C and -3×10^{-8}C are located 16cm apart. At what point on the line joining the two charges is the electric potential zero?

Sol. Let P be a point on the line joining the two charges where the potential is zero.



If the potential due to the charge q_1 and q_2 is zero, then

$$V_1 + V_2 = 0$$

$$\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2} = 0$$

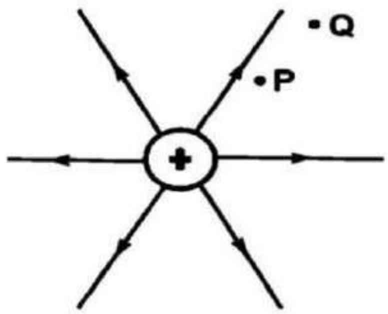
$$\frac{5\times10^{-8}}{x} - \frac{3\times10^{-8}}{16-x} = 0$$

or $$\frac{5}{x} - \frac{3}{16-x} = 0$$

$$\frac{5}{x} = \frac{3}{16-x} \quad \Rightarrow \quad 5(16-x) = 3x$$

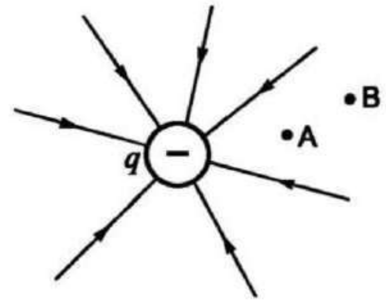
$$x = 10\text{cm}$$

Q. CBSE 2014 Fig. Shows the field lines on a positive charge. Is the work done by the field in moving a small positive charge from Q to P positive or negative?



Sol. The work done by the field is negative. This is because the charge is moved against the force exerted by the field.

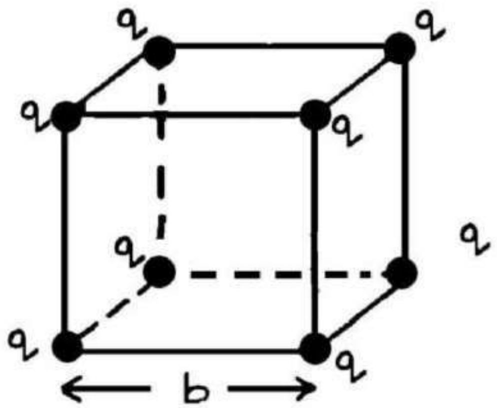
Q. (CBSE 2015) The field lines of a negative point charge are as shown in fig. Does the Kinetic energy of a small negative charge increase or decrease in going from B to A ?



Sol. The Kinetic energy of a negative charge decreases while going from point B to point A, against the movement of force of repulsion.

Q. A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Sol. The length of diagonal of the cube of each side b is $= \sqrt{3}\, b$



$\therefore$ Distance between centre of cube and each vertex is $r = \sqrt{3}\, b/2$

Here 8 charges each of value q are present at the 8 vertices of the cube, therefore total potential is

$$V = V_1 + V_2 + \dots V_8$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{8q}{\sqrt{3}b/2} = \frac{4q}{\sqrt{3}\pi\epsilon_0 b}$$

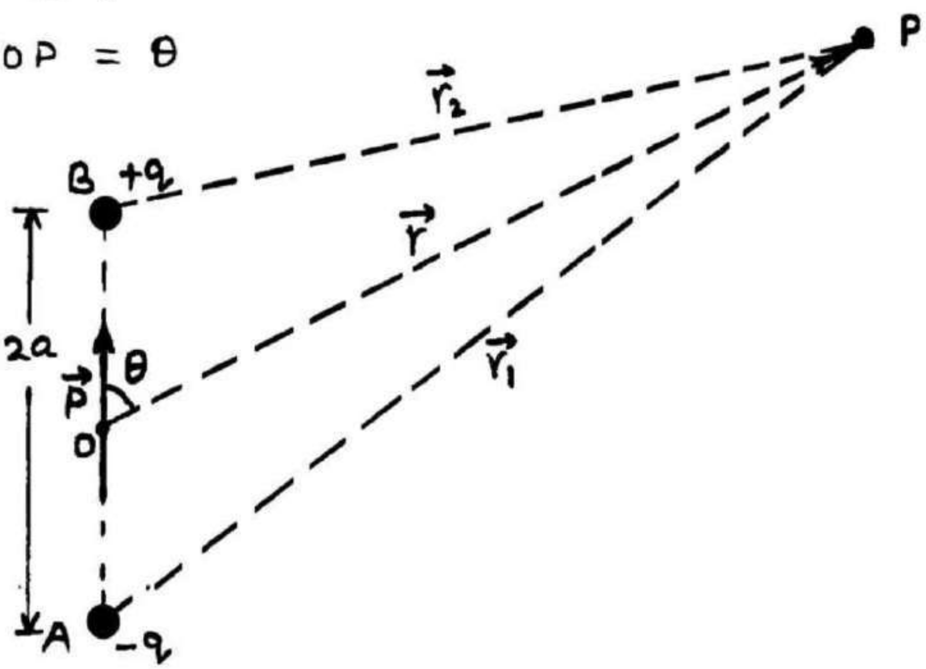
Further electric field at the centre due to all the eight charges is zero, because the field due to individual charges cancel in pairs.

Electrostatic Potential due to an Electric Dipole :-

Let us take the origin at the centre of the dipole. Now we have to calculate electric potential at point P

where $\vec{OP} = \vec{r}$

and $\angle BOP = \theta$



Let the distance of P from charge $-q$ at A be r_1 and distance of P from charge $+q$ at B is r_2.

Electrostatic potential at P due to $-q$ and $+q$ charge is

$$V_1 = \frac{-q}{4\pi\epsilon_0 r_1} \quad \text{and} \quad V_2 = \frac{q}{4\pi\epsilon_0 r_2}$$

So the potential at P due to the dipole is -

$$V = V_1 + V_2$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \quad \text{--- (1)}$$

Now by geometry -

$$\cos\theta = \frac{a^2 + r^2 - r_2^2}{2ra} \quad (\text{for } \Delta BOP)$$

or $$r_2^2 = r^2 + a^2 - 2ra\cos\theta$$

Similarly $$\cos(180^\circ - \theta) = \frac{a^2 + r^2 - r_1^2}{2ra} \quad \text{(for triangle AOP)}$$

or $$r_1^2 = r^2 + a^2 + 2ra\cos\theta$$

Here we may write-

$$r_1^2 = r^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)$$

If $a << r$ then $\frac{a}{r}$ is small and $\frac{a^2}{r^2}$ can be neglected.

$$\therefore \quad r_1^2 = r^2\left(1 + \frac{2a}{r}\cos\theta\right)$$

$$r_1 = r\left(1 + \frac{2a}{r}\cos\theta\right)^{1/2}$$

and $$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{2a}{r}\cos\theta\right)^{-1/2}$$

Similarly $$\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-1/2}$$

so by equation (1) -

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-1/2} - \frac{1}{r}\left(1 + \frac{2a}{r}\cos\theta\right)^{-1/2}\right]$$

Using Binomial theorem we get-

$$V = \frac{q}{4\pi\epsilon_0 r}\left[\left(1 + \frac{a}{r}\cos\theta\right) - \left(1 - \frac{a}{r}\cos\theta\right)\right]$$

$$V = \frac{q \times 2a\cos\theta}{4\pi\epsilon_0 r^2}$$

$$V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2} \quad [\text{As } P = q \times 2a]$$

As $P\cos\theta = \vec{P}\cdot\hat{r}$, where $\hat{r}$ is unit vector along the position vector $\vec{OP} = \vec{r}$

So the electrostatic potential at P due to a dipole is -

$$\boxed{V = \frac{\vec{p} \cdot \hat{r}}{4\pi\epsilon_0 r^2}}$$

(i) <u>On the dipole axis $(\theta = 0^\circ \text{ or } \pi)$</u>

$$\boxed{V = \pm \frac{p}{4\pi\epsilon_0 r^2}}$$

Positive sign for $\theta = 0^\circ$ and negative sign for $\theta = \pi$.

(ii) <u>On the equatorial plane</u> $\left(\theta = \frac{\pi}{2};\ \cos\frac{\pi}{2} = 0\right)$

$$\boxed{V = 0}$$

So Electrostatic potential at any point in the equatorial plane of dipole is zero.

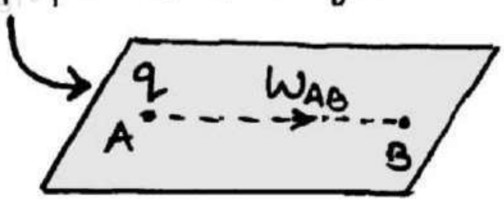
The potential due to a dipole is axially symmetric about $\vec{p}$.

Equipotential Surfaces

That surface at every point of which electric potential is the same is called an equipotential surface.

(i) No work is done on moving the test charge from one point to the other on equipotential surface.

equipotential surface



$V_A = V_B$

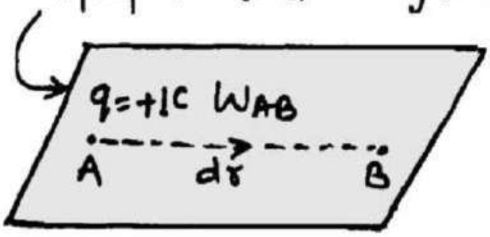
by $\frac{W_{AB}}{q} = V_B - V_A$

$\frac{W_{AB}}{q} = 0$

$\boxed{W_{AB} = 0}$

(ii) The equipotential surface through a point is normal to the electric field at that point.

equipotential surface



$V_A = V_B$

$W_{AB} = 0$ — ①

Also $W_{AB} = \vec{F} \cdot \vec{dr}$

$W_{AB} = F dr \cos\theta$

Here $q = +1C$ so that

by $\vec{F} = q\vec{E} \Rightarrow \vec{F} = \vec{E}$

Hence $W_{AB} = E dr \cos\theta$ — ②

by eq. ① and ②

$E dr \cos\theta = 0$

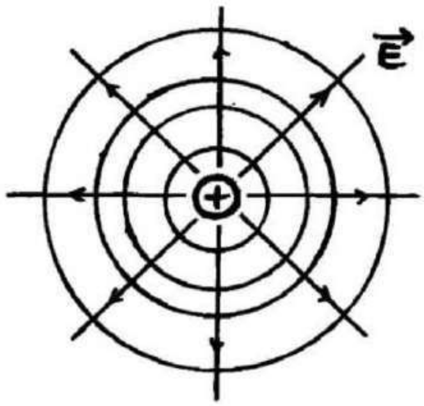
$\theta = 90^\circ$

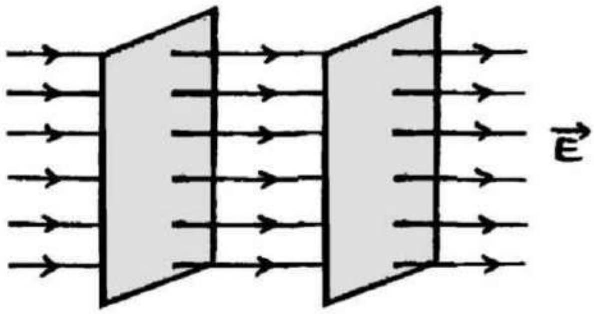
(iii) Two Equipotential surfaces never intersect each other.

* The equipotential surface of a single point charge are concentric spherical surfaces centred at the charge.

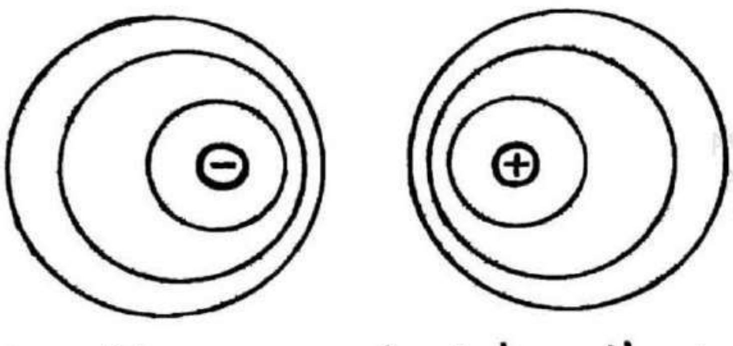
* For a uniform electric field say along x-axis, the equipotential surfaces are planes normal to the x axis.

Some Important Equipotential surfaces





(a) For a point charge (b) For uniform electric field.



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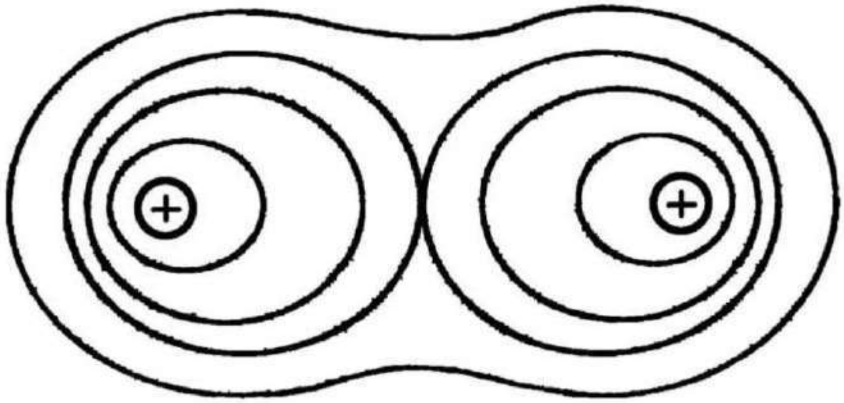
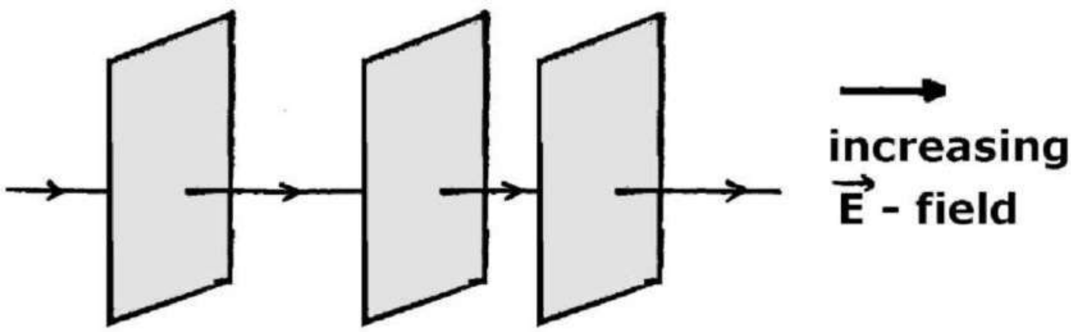
(c) For an electric dipole.



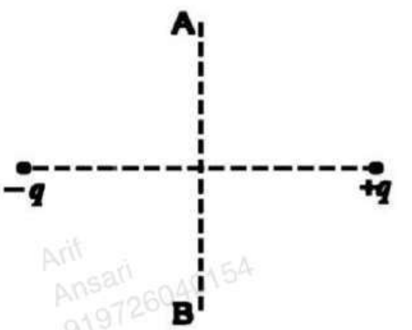
(d) For two identical Positive charges



(e) For nonuniform electric field.

Q. (CBSE 2016) A charge is moved from a point A to B in equatorial plane of a dipole without acceleration find the work done in the process.

Sol. The equatorial plane of a dipole is a equipotential surface so the work done in moving charge will be zero.



$$V_B - V_A = \frac{W_{AB}}{q}$$

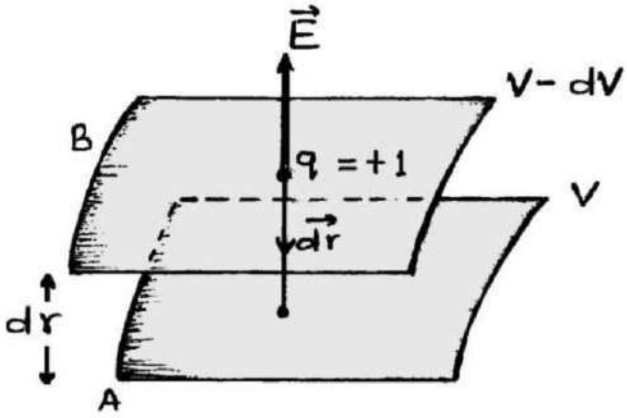
$\Rightarrow$ Here $V_B = V_A$

$$W_{AB} = 0$$

Relation between Electric Field and Potential

Let us consider two equipotential surfaces A and spaced closely at a perpendicular distance $\vec{dr}$.

Let the potential of A be $V_A = V$ and potential of be $V_B = V - dv$ where dv is decrease in potential in the direction of electric field $\vec{E}$ normal to A and B.



A unit positive charge ($q = +1$) is taken along this perpendicular distance from the surface B to the surface A against the electric field. So work done

$$W_{BA} = \vec{F} \cdot \vec{dr}$$

$$W_{BA} = -E\,dr \qquad (\text{as } \vec{F} = q\vec{E})$$

$$W_{BA} = V_A - V_B$$

$$= V - (V - dV) = dv$$

$$\text{Hence} \quad -E\,dr = dv \qquad \text{or} \qquad \boxed{E = -\frac{dv}{dr}}$$

Negative sign shows that the direction of electric field $\vec{E}$ is in the direction of decreasing potential.

* The magnitude of electric field is given by change in magnitude of potential per unit displacement normal to the equipotential surface at the point. This is called potential gradient.

$$|\vec{E}| = -\frac{|dv|}{dr} = -(\text{Potential gradient})$$

Q. Three points A, B and C lie in a uniform electric field of $E = 5\times10^{3}$ N/C as shown in the figure. Find the potential difference between A and C.

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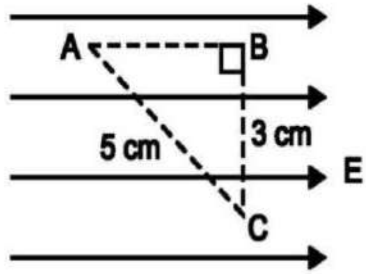


Sol. The line joining B to C is perpendicular to electric field, so potential of B = Potential of C

i.e. $V_B = V_C$

Distance $AB = 4$ cm

Potential difference between A and C $= E \times AB$

$$= 5\times10^{3}\times 4\times10^{-2}$$

$$= 200 \text{ Volt.}$$

BE A WARRIOR
— NOT —
A WORRIER

Electrostatic Potential Energy of a System of charges :-

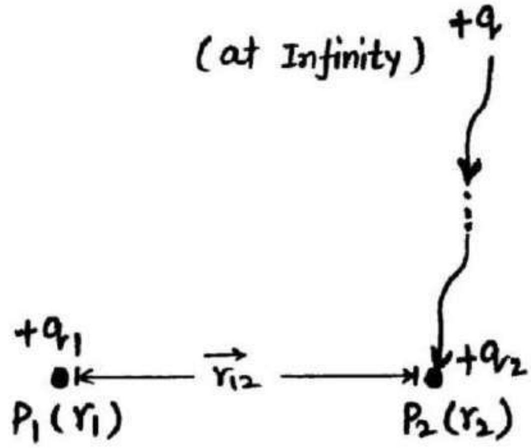
Electrostatic potential energy of a system of point charges is the total amount of work done in bringing the various charges to their respective positions from infinitely large mutual separation.

(i) For a system of two point charges

Suppose a point charge q_1 is held at a point P_1 with position vector $\vec{r}_1$ in space. Another point charge q_2 is at infinite distance from q_1. This is to be brought to the position P_2 with position vector $\vec{r}_2$.

In bringing the first charge q_1 to position $P_1(\vec{r}_1)$ no work is done, because all other charges are still at infinity, and there is no field.

i.e. $\boxed{W_1 = 0}$



Now, work done in carrying charge q_2 from ∞ to P_2

$$W_2 = [\text{Potential due to } q_1] \times q_2$$

$$W_2 = \frac{q_1}{4\pi\epsilon_0 r_{12}} \times q_2$$

Total work done $W = W_1 + W_2$

$$\boxed{\text{Electrostatic potential energy } U = W = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}}$$

(ii) For a system of N point charges -

It is equal to the total amount of work done in assembling all the charges at the given positions from infinity.

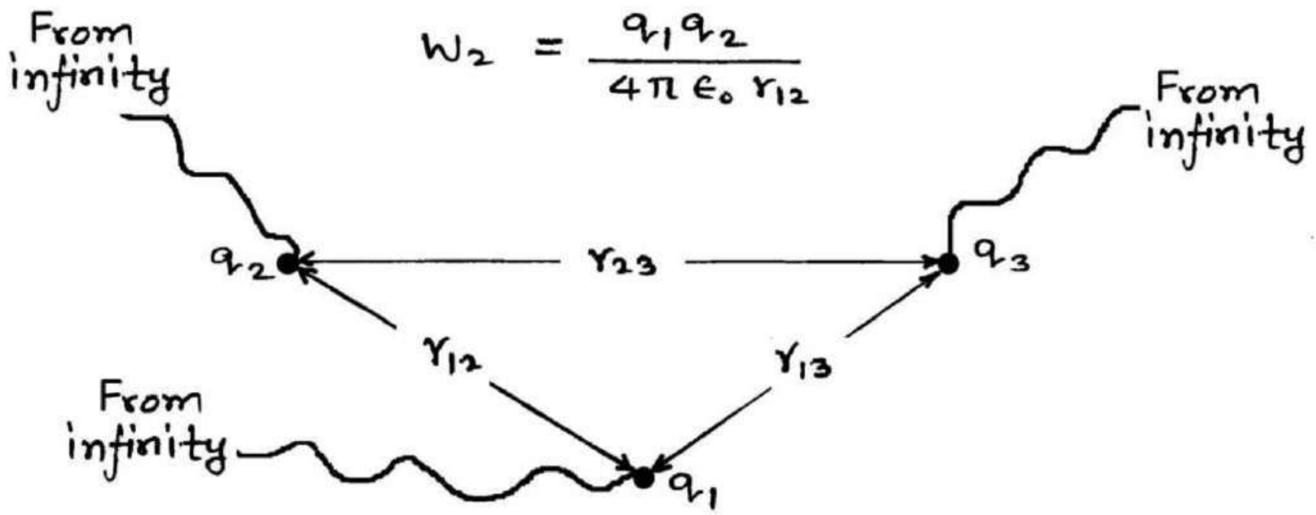
In bringing the first charge q_1 to position $P_1(\vec{r}_1)$ no work is done.

i.e. $W_1 = 0$

When we bring charge q_2 from infinity to $P_2(\vec{r}_2)$ at a distance r_{12} from q_1, work done is

$$W_2 = [\text{Potential due to } q_1] \times q_2$$

$$W_2 = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$



In bringing q_3 from ∞ to $P_3(\vec{r}_3)$, work has to be done against electrostatic forces due to both q_1 and q_2

$$\therefore \quad W_3 = [\text{Potential due to } q_1 \text{ and } q_2] \times q_3$$

$$W_3 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right) \times q_3$$

$\therefore$ Electrostatic potential energy of a system of three charges

$$U = W_1 + W_2 + W_3 = 0 + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} + \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$\boxed{U = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)}$$

Proceding in this way, we can write electrostatic potential energy of a system of N point charges at $\vec{r}_1, \vec{r}_2, \ldots \vec{r}_N$ as -

$$\boxed{U = \frac{1}{4\pi\epsilon_0} \sum_{\text{All Pairs}} \frac{q_j q_k}{r_{jk}}}$$

Potential Energy of a Single charge in an external Field :-

If $V(\vec{r})$ is external potential at any point P of position vector $\vec{r}$ then work done in bringing a charge q from infinity to the point P in the external field is

$$W = q \cdot V(\vec{r})$$

This work is stored in the charged particle in the form of its potential energy.

$$\boxed{\text{Potential energy of a charge } q\ (U) = q \cdot V(\vec{r})}$$

Potential Energy of a System of two Charges in an external Field :-

Suppose q_1 and q_2 are two point charges at position vectors $\vec{r}_1$ and $\vec{r}_2$ respectively in a uniform external electric field $\vec{E}$.

Now work done in bringing charge q_1 and q_2 from infinity to position $\vec{r}_1$ and $\vec{r}_2$ are

$$\boxed{W_1 = q_1 \cdot V(\vec{r}_1)} \quad \text{and} \quad \boxed{W_2 = q_2 \cdot V(\vec{r}_2)}$$

While bringing q_2 from infinity to position $\vec{r}_2$, work has also to be done against the field due to q_1. This is.

$$\boxed{W_3 = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}}$$

where r_{12} is the distance between q_1 and q_2.

So Potential energy of the system = total work done

$$U = W_1 + W_2 + W_3$$

$$\boxed{U = q_1 \cdot V(\vec{r}_1) + q_2 \cdot V(\vec{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}}$$

[redacted]

Q. (a) Determine the electrostatic potential energy of a system consisting of two charges 7 μC and −2 μC (with no external field) placed at (−9 cm, 0, 0) and (9 cm, 0, 0) respectively.
(b) How much work is required to separate the two charges infinitely away from each other?
(c) Suppose that the same system of charges is now placed in an external field $E = A \times \frac{1}{r^2}$, where $A = 9 \times 10^5\ C/m^2$. What would be the electrostatic energy of the configuration?

Sol. (a)
$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

$$= \frac{7\times10^{-6} \times (-2\times10^{-6}) \times 9\times10^{9}}{(9+9)\times10^{-2}} = -0.7\ J$$

(b) Work required $= U_2 - U_1 = 0 - U$

$= 0.7\ J$

(c) In the electric field total P.E. $= \frac{q_1 q_2}{4\pi\epsilon_0 r} + q_1 V(\vec{r}_1) + q_2 V(\vec{r}_2)$

from $E = -\frac{dV}{dr}$; $V = \int -E\,dr$

$$= \int -\frac{A}{r^2}\,dr = \frac{A}{r}$$

$\therefore$ Total P.E. $= \frac{q_1 q_2}{4\pi\epsilon_0 r} + \frac{A q_1}{r_1} + \frac{A q_2}{r_2}$

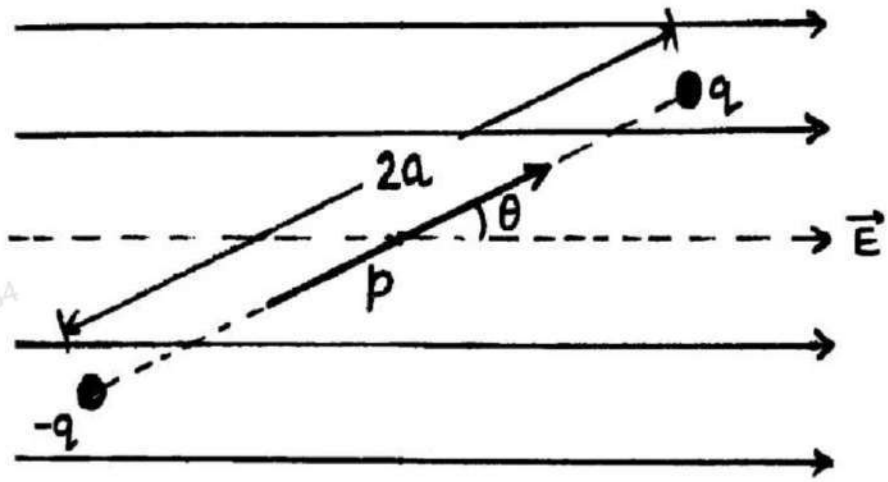
$$= -0.7 + \frac{9\times10^5 (7\times10^{-6})}{0.09} + \frac{9\times10^5 (-2\times10^{-6})}{0.09}$$

So Total P.E. $= -0.7 + 70 - 20$

$= 49.3\ J$

Potential Energy of a dipole in external Field

Consider a dipole placed in a uniform electric field E.



Here the dipole experiences no net force but experience a torque τ given by -

$$\boxed{\tau = \vec{P} \times \vec{E} = PE \sin\theta}$$

Which will tend to rotate it (unless $\vec{P}$ is parallel to $\vec{E}$).

Suppose an external torque τ_{ext} is applied in such a manner that it just neutralises this torque and rotate the dipole from angle θ_1 to angle θ_2 without angular acceleration.

The amount of work done by the external torque will be given by -

$$W = \int_{\theta_1}^{\theta_2} \tau_{ext}(\theta)\, d\theta = \int_{\theta_1}^{\theta_2} PE \sin\theta \, d\theta$$

$$\boxed{W = -PE(\cos\theta_2 - \cos\theta_1)}$$

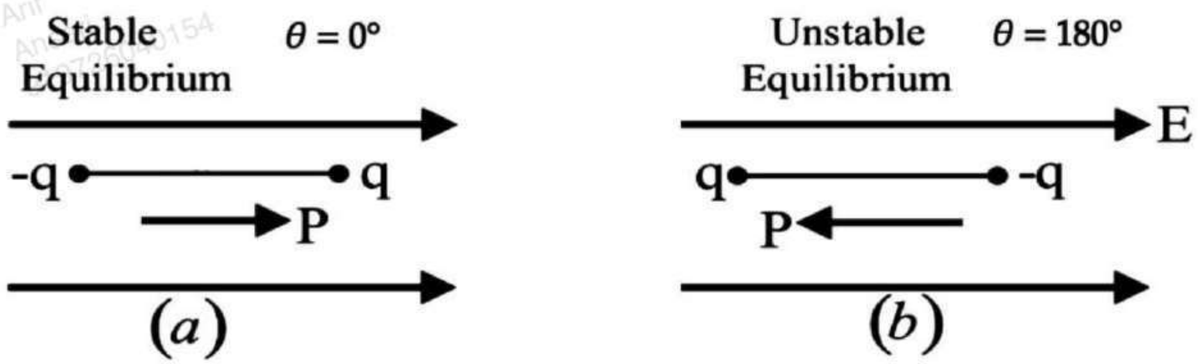
This work is stored as the potential energy of the system.

If we take $\theta_1 = \pi/2$ where the potential energy U is taken to be zero.

So $$U(\theta_2) = -PE\left(\cos\theta_2 - \cos\frac{\pi}{2}\right)$$

$\therefore$ $$\boxed{U(\theta) = -PE\cos\theta = -\vec{P}\cdot\vec{E}}$$

Special cases:



Electrostatics of Conductors

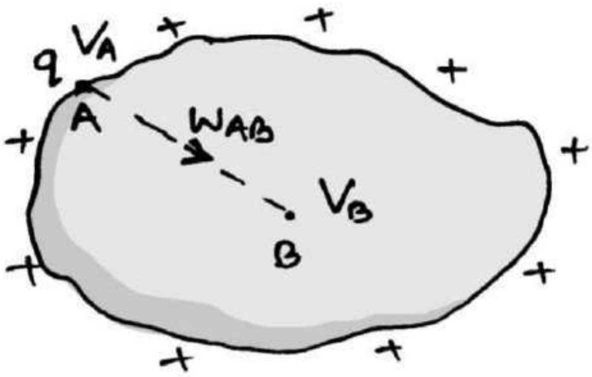
(1) Electrostatic field is zero inside a conductor.

(2) At the surface of a charged conductor, electrostatic field must be normal to the surface at every point.

If E were not normal to the surface, it would have some non-zero components along the surface. Free charges on the surface of the conductor would then experience force and move. In the static situation therefore E should have no tangential component.

(3) There is no excess charge at any point inside the conductor than the static situation, and any excess charge must reside at the surface.

(4) Electrostatic potential is constant throughout the volume of the conductor and has the same value as inside on its surface.



by $\dfrac{W_{AB}}{q} = V_B - V_A$

Here inside the conductor $E = 0$

So that $W_{AB} = 0$

Hence $\dfrac{0}{q} = V_B - V_A$

$V_B - V_A = 0$

$$\boxed{V_B = V_A}$$

(5) Electric field at the surface of a charged conductor is

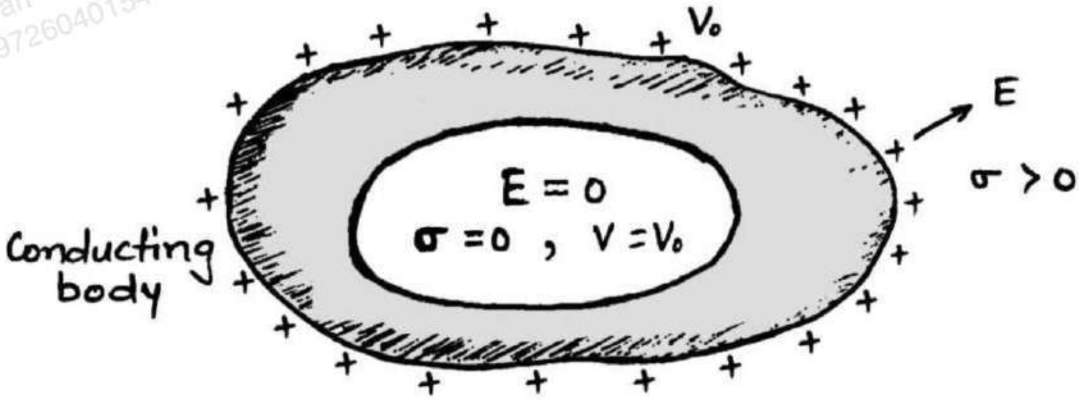
$$\boxed{\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}}$$

Here $\hat{n}$ is unit vector normal to the surface and σ is surface charge density.

Electrostatic Shielding

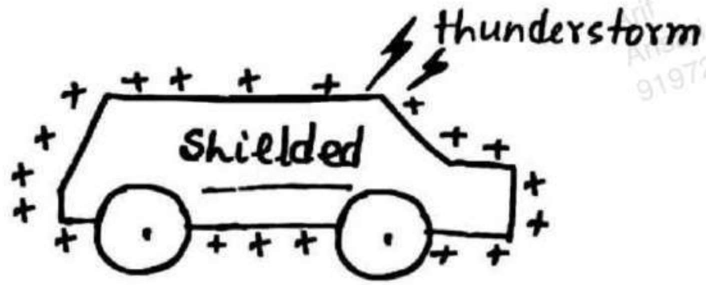
Electrostatic shielding / screening is the phenomenon of protecting a certain region of space from external electric field.

We know that the electric field inside a conductor is zero. Therefore to protect delicate instruments from external electric fields, we enclose them in hollow conductors. Such hollow conductors are called Faraday cages. They need not be earthed.



The fundamental fact is that electric field inside a cavity is zero, whatever be the size and shape of the cavity and whatever be the charge on the conductor and the external field in which it might be placed.

In thunderstorm accompained by lightening, it is safer to be inside a car or bus then to be in the open ground or under a tree etc. The metallic body of the car or bus provides electrostatic shielding from the lightening.



Non polar and Polar Dielectrics

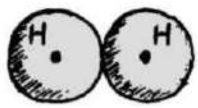
Dielectrics are insulating materials which transmit electric effects without actually conducting electricity.

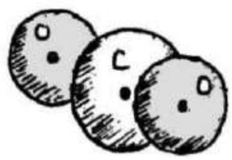
Dielectrics
- Non polar
- Polar

(a) Non Polar Dielectrics

The molecules in which the centre of positive charge coincide with the centre of negative charge are called non-polar molecules.
Such molecules are symmetric in shape.

Examples - O_2, CO_2, CH_4 etc.

H_2 

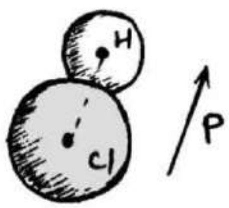


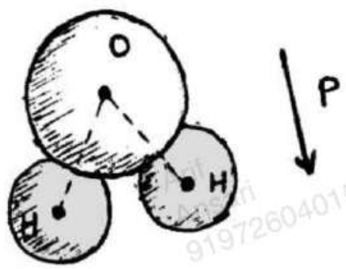
CO_2

(b) Polar Dielectrics

If the centres of positive and negative charges do not coincide because of the asymmetric shape of the molecules, then these are called polar molecules.

Examples - HCl, H_2O, NH_3 etc.

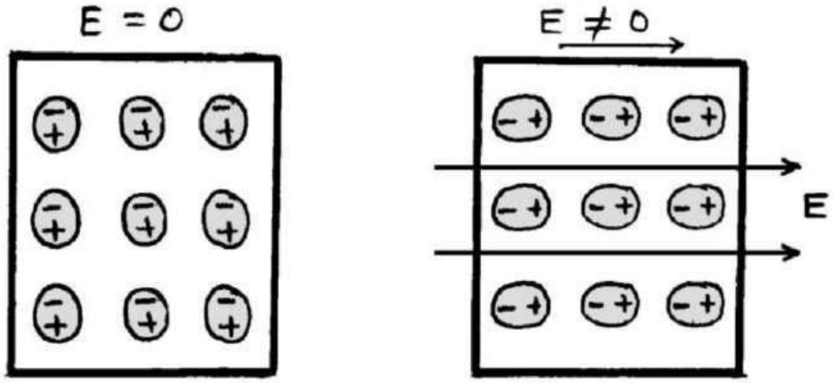




Dielectric Polarization

When a non-polar dielectric is held in an external electric field $\vec{E}$, the centre of positive charge (protons) in each molecule is pulled in the direction of $\vec{E}$ and the centre of negative charge (electrons) is pulled in a direction apposite to $\vec{E}$.

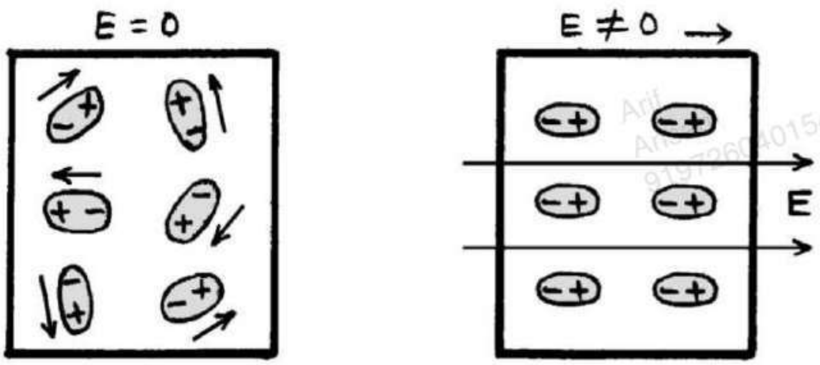
Therefore the two centres of positive and negative charges in the molecule are separated. The molecule gets distorted and is said to be polarised.



Non-Polar Molecules

When no external field is applied to a dielectric with polar molecules then the different permanent dipoles are oriented randomly due to thermal agitation, so the total dipole moment is zero.

When an external electric field is applied, the individual dipole moments tends to align with the field. When summed over all the molecules, there appears some net dipole moment in the direction of the external field, i.e. the dielectric is polarised.



Polar Molecules

Thus in either case whether polar or nonpolar a dielectric develops a net dipole moment in the presence of an external field the dipole moment per unit volume is called polarisation and is denoted by P.

Electric Susceptibility

It is found that the electric polarization P is directly proportional to the effective electric field (E) in a polarized dielectric.

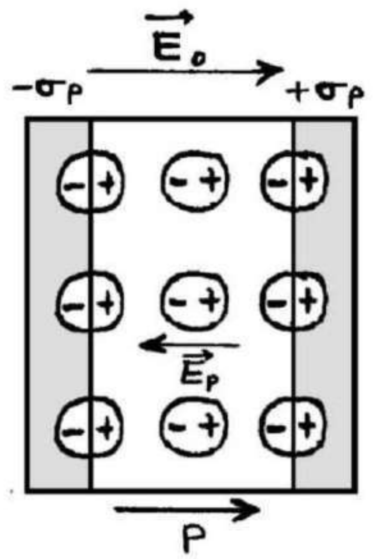
$$P \propto E$$

or

$$\boxed{P = \chi E}$$

where χ is a constant and is pronounced as 'chi'.

- χ is called electric susceptibility of the dielectric.
- The electric susceptibility describes the electrical behaviour of a dielectric.
- For vacuum $\boxed{\chi = 0}$



Effective Electric field in a polarized dielectric is -

$$\boxed{E = E_0 - E_p}$$

Q. (CBSE 2005) Why does the electric field inside a dielectric decrease when it is placed in an external electric field?

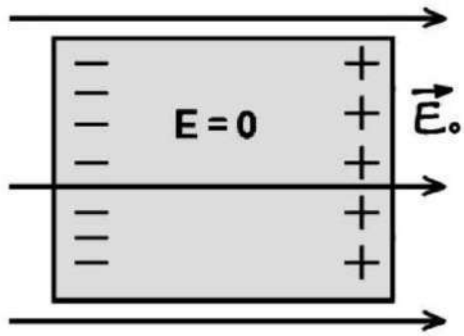
Sol. When a dielectric is placed in an external field, the field causes polarisation of the dielectric inducing a field in the opposite direction. Thus the net electric field inside the dielectric decreases.

Q. (a) Explain briefly using a proper diagram, the difference in behaviour of a conductor and a dielectric in the presence of external electric field.

CBSE 2019

(b) Define the term polarisation of a dielectric, and write expression for a linear isotropic dielectric in terms of electric field.

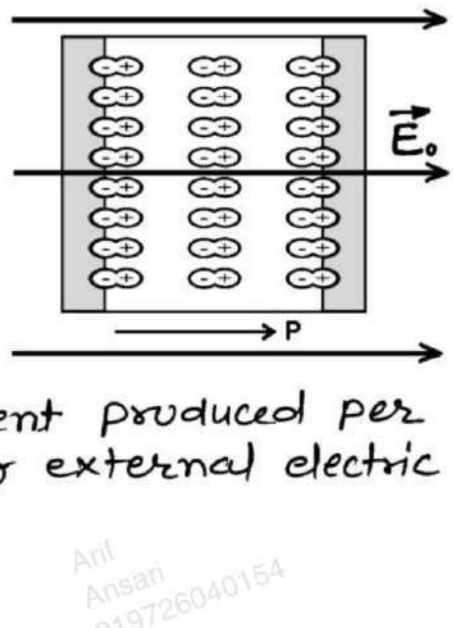
Sol. (a) **For Conductor:**



Due to electrostatic induction the free electrons collect on the left face of the slab creating equal positive charge on the right face.

Internal electric field is equal to the external field, hence net electric field inside the conductor is zero.

For Dielectric:



Due to alignment of atomic dipoles along external electric field, the net electric field within the dielectric decreases.

$$E = E_0 - E_p$$

(b) The net electric dipole moment produced per unit volume in the presence of external electric field is called polarisation $\vec{P}$.

$$\vec{P} = \chi \vec{E}$$

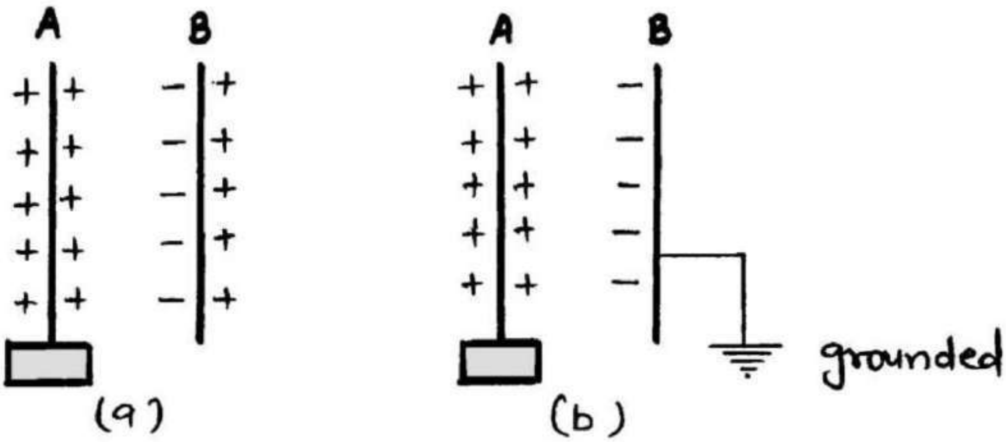
χ = Electrical Susceptibility

Capacitors and Capacitance

A capacitor is a system of two conductors separated by an insulator. It is an arrangement for storing large amounts of electric charge and hence electrical energy in a small space.

Principle of Capacitor

Lets consider an insulated metal plate A. Let some positive charge be given to this plate, till its potential become maximum.



Now consider another insulated metal plate B held near the plate A. By induction a negative charge is produced on the nearer face of B and an equal positive charge develops on the farther face of B.

Now connect the plate B to earth. The induced positive charge on B being free, flow to earth. The induced negative charge on B, however stays on as it is bound to positive charge on A.

Due to the induced negative charge on B, potential of A is greatly reduced. Thus a Large amount of charge can be given to A to raise it to the maximum potential.

Hence we conclude that the capacitance of an insulated conductor is increased considesably by bringin near it an uncharged earthed conductor.

Electrical Capacitance

The ability of a conductor to store electric charge is called electrical capacitance

If a charge Q given to the conductor raises its potential by V then it is found that

$$Q \propto V \qquad \text{or} \qquad Q = CV$$

where c is a constant called electrical capacitance.

$$\boxed{C = \frac{Q}{V}}$$

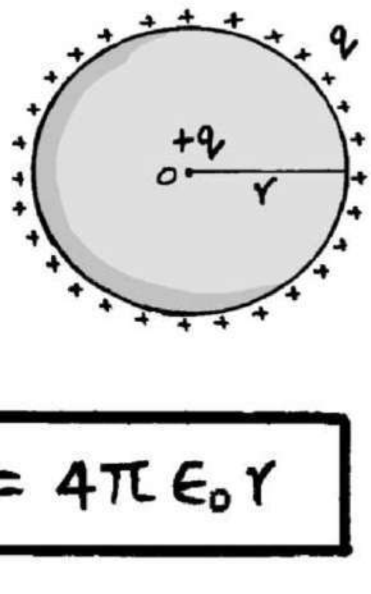
SI unit of capacitance - Farad

A conductor is said to have a capacity of 1 farad, when a charge of 1 coulomb raises its potential by 1 volt.

Capacity of an Isolated Spherical Conductor

Let a charge +q is given to a sphere.

As the sphere behaves as if the entire charge is concentrated at the centre of the sphere, so the potential at any point on the surface of the sphere is



$$V = \frac{q}{4\pi\epsilon_0 r}$$

As $C = \frac{q}{V}$ so

$$\boxed{C = 4\pi\epsilon_0 r}$$

Capacity of earth

We can calculate electrical capacity of earth, taking it to be a conducting sphere of radius $r = 6400$ km.

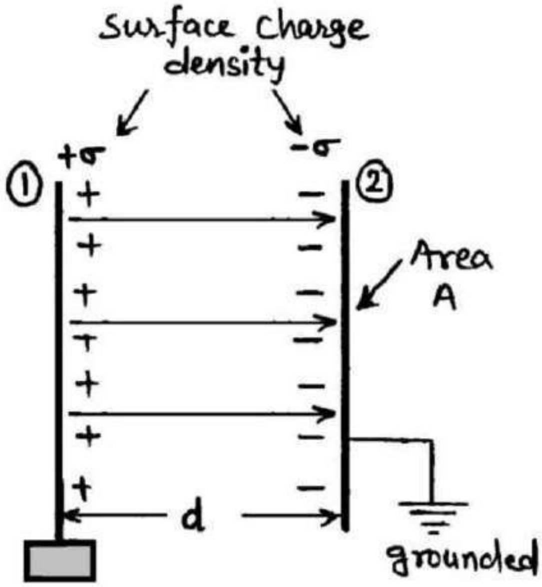
$$C = 4\pi\epsilon_0 r = \frac{1}{9 \times 10^9} \times 6.4 \times 10^6 = 711\ \mu F$$

Thus the capacity of earth is only 711 μF.

Parallel Plate Capacitor

A parallel plate capacitor consist of two thin conducting plates 1 and 2, each of area A held parallel to each other, suitable distance d apart.

The plates are separated by an insulating medium like air, paper, mica, glass etc.



When a charge +Q is given to the insulated plate 1, then a charge -Q is induced on the nearer face of plate 2,

surface charge density of plate 1 and 2 are σ and $-\sigma$.

In the region between the plates, separated by air / vacuum, electric field intensity is

$$E = \frac{\sigma}{\epsilon_0} = \frac{1}{\epsilon_0} \times \frac{Q}{A}$$

If E is the electric field between the plates, than the potential difference V between the plates is -

$$V = E \times d = \frac{1}{\epsilon_0} \times \frac{Q}{A} d$$

So the capacity of parallel plate capacitor is -

$$C = \frac{Q}{V} \Rightarrow C = \frac{Q}{\frac{1}{\epsilon_0} \times \frac{Q}{A} d}$$

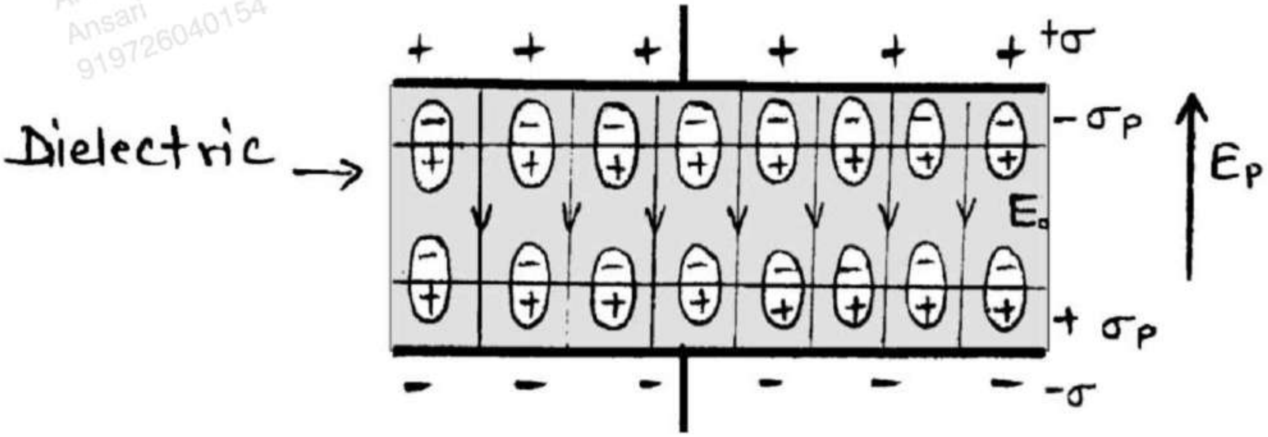
$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

Effect of Dielectric on capacitance

When there is a vacuum between the plates of a capacitor having the charge $\pm Q$ on its plates with charge density $\pm\sigma$ then -

$$E_0 = \frac{\sigma}{\epsilon_0} , \quad V_0 = E_0 d , \quad C_0 = \frac{Q}{V_0} = \frac{\epsilon_0 A}{d}$$

Now consider a dielectric is inserted between the plates which fully occupying the intervening region.



The dielectric is polarised by the field with surface charge densities σ_p and $-\sigma_p$ at the surface of the dielectric. So the net surface charge density on the plates is $\pm(\sigma - \sigma_p)$.

So the final electric field inside the dielectric is -

$$E = \frac{\sigma - \sigma_p}{\epsilon_0}$$

and the potential difference between the plates is -

$$V = Ed = \frac{\sigma - \sigma_p}{\epsilon_0} \times d$$

For linear dielectrics, we expect σ_p to be proportional to E_0, i.e. to σ. Thus $(\sigma - \sigma_p)$ is proportional to σ and we can write -

$$\sigma - \sigma_p \propto \sigma$$

$$\sigma - \sigma_p = \frac{\sigma}{K}$$

$K \rightarrow$ dielectric Constant and $K > 1$

So

$$\boxed{V = \frac{\sigma d}{\epsilon_0 K} = \frac{Q d}{A \epsilon_0 K}}$$

So the Capacitance C with dielectric between the plates is -

$$C = \frac{Q}{V}$$

$$= \frac{Q}{\frac{Q d}{A \epsilon_0 K}} = \frac{K \epsilon_0 A}{d}$$

Or

$$\boxed{C = \frac{K \epsilon_0 A}{d}}$$

Here $K = \frac{\epsilon}{\epsilon_0}$ is Called the dielectric Constant of the substance.

So that $\boxed{C = K C_0}$

- When dielectric is introduced, the field between the plates of the capacitor decreases by K. Due to which V decreases by a factor of K. This implies that the capacitance increase by K.

- Dielectric constant of a medium is defined as the ratio of the capacitance of the capacitor with the dielectric as the medium to its capacitance with vacuum between its plates.

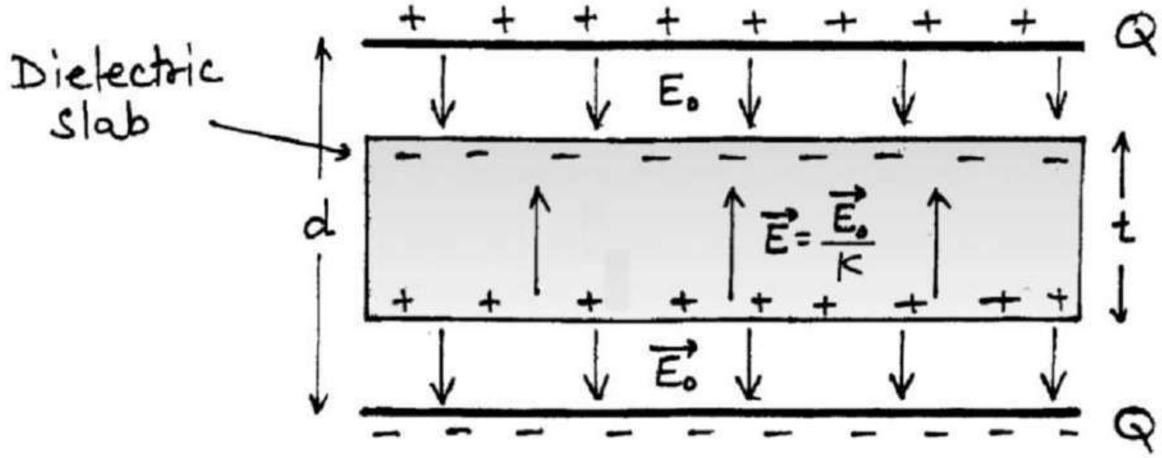
$$\boxed{K = \frac{C}{C_0}}$$

Capacitance of a Parallel plate Capacitor with a dielectric slab -

The Capacitance of a parallel plate Capacitor of plate area A and plate separation d with vacuum or air inbetween is -

$$\boxed{C_0 = \frac{\epsilon_0 A}{d}}$$

Suppose $\pm Q$ are the charges on the Capacitor plates which produce a uniform electric field E_0 in the space between the plates.



When a dielectric slab of thickness $t < d$ is introduced between the plates, the molecules in the slab get polarized in the direction of E_0.

Therefore the effective field inside the dielectric is

$$E = \frac{E_0}{K}$$

outside the dielectric field remains E_0 only.

Therefore, potential difference between the two plates is

$$V = E_0(d-t) + Et$$

$$V = E_0(d-t) + \frac{E_0}{K}t$$

$$V = E_0\left[d - t + \frac{t}{K}\right]$$

As $$E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

$$\therefore \quad V = \frac{Q}{A\epsilon_0}\left[d - t + \frac{t}{K}\right]$$

$\therefore$ Capacitance of the capacitor with dielectric inbetween is

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{A\epsilon_0}\left[d - t + \frac{t}{K}\right]}$$

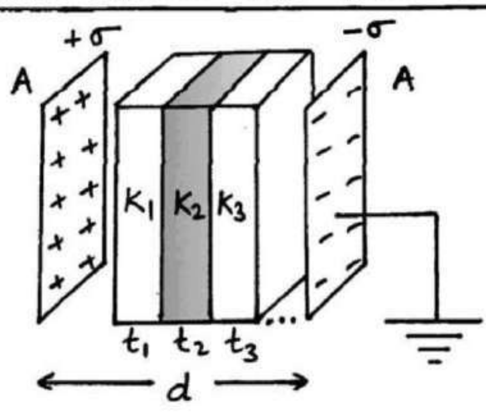
$$\boxed{C = \frac{\epsilon_0 A}{d - t + \frac{t}{K}}}$$

Here $C > C_0$

* Hence on introducing a dielectric slab inbetween the plates of a capacitor, its capacitance increases.

Case 1. When a number of dielectric slabs are inserted between the plate

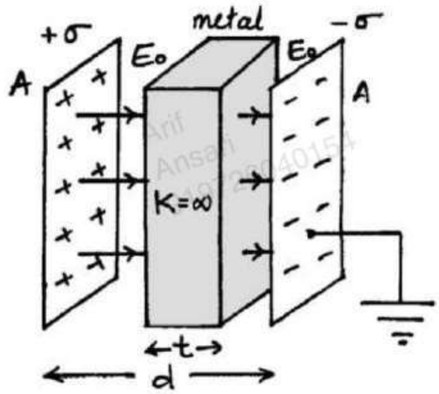
$$\boxed{C = \frac{\epsilon_0 A}{d - (t_1 + t_2 + \ldots) + \left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \ldots\right)}}$$



Case 2. When a metallic slab is inserted between the plates

$$C = \frac{\epsilon_0 A}{\left[d - t + \frac{t}{\infty}\right]}$$

$$\boxed{C = \frac{\epsilon_0 A}{(d - t)}}$$



Q. The two plates of a parallel plate capacitor are 4 mm apart. A slab of dielectric constant 3 and thickness 3mm is introduced between the plates with its faces parallel to them. The distance between the plates is so adjusted that the capacitance of the capacitor become 2/3 of its original value. What is the new distance between the plates?

CBSE 2008

Sol. Here distance between parallel plates is $d = 4\,mm$

$$d = 4 \times 10^{-3}\,m.$$

$K = 3$, thickness $t = 3\,mm = 3 \times 10^{-3}\,m.$

$$\therefore \quad C = \frac{\epsilon_0 A}{d} \quad \text{and} \quad C_1 = \frac{\epsilon_0 A}{d_1 - t\left(1 - \frac{1}{K}\right)}$$

Since $\quad C_1 = \frac{2}{3} C.$

$$\therefore \quad \frac{\epsilon_0 A}{d_1 - t\left(1 - \frac{1}{K}\right)} = \frac{2}{3}\frac{\epsilon_0 A}{d}$$

or

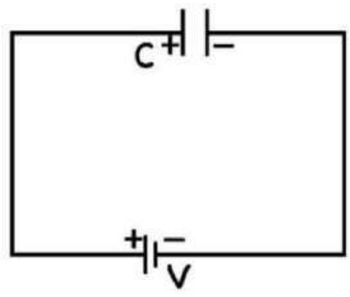
$$\frac{1}{d_1 - t\left(1 - \frac{1}{K}\right)} = \frac{2}{3d}$$

$$\frac{1}{d_1 - 3 \times 10^{-3}\left(1 - \frac{1}{3}\right)} = \frac{2}{3 \times 4 \times 10^{-3}}$$

So $\quad d_1 = 8 \times 10^{-3}\,m. = 8\,mm.$

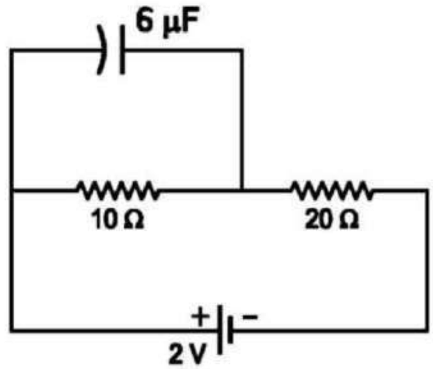
Q. CBSE 2017 Why does current in a steady state not flow in a capacitor connected across a battery? However momentary current does flow during charging or discharging of the capacitor. Explain.

Sol. (i) In the steady state no current flows through capacitor because, we have two sources (battery and fully charged capacitor) of equal potential connected in opposition.



(ii) During charging or discharging there is a momentary flow of current as the potentials of the two sources are not equal to each other.

Q. CBSE 2014, 18 Find the charge on the capacitor as shown in the circuit.



Sol. In steady state when the capacitor is fully charged, no current flows through it.

Total resistance $R = 10 + 20 = 30\Omega$

then current $I = \frac{V}{R}$

$$I = \frac{2}{30} = \frac{1}{15} A$$

Potential difference $V = IR$

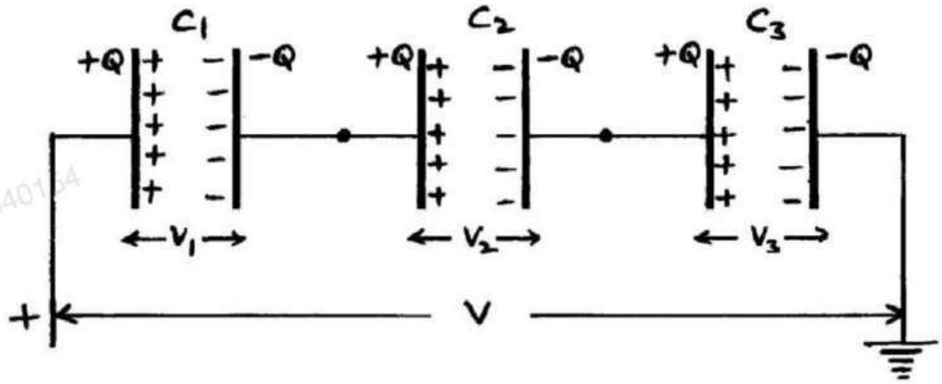
$$V = \frac{1}{15} \times 10 = \frac{2}{3} V$$

Charge $Q = CV \Rightarrow Q = 6 \times \frac{2}{3} = 4 \mu C$

Combination of Capacitor

(i) Capacitors in Series

Three capacitors of capacities C_1, C_2 and C_3 are connected in series. V is potential difference applied across the combination.



In series combination each capacitor receives the same charge of Q. As there capacities are different, so the potential differences across the three capacitors are different.

$$V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}, \quad V_3 = \frac{Q}{C_3}$$

If C_s is the total capacitance of combination than

$$V = \frac{Q}{C_s}$$

As total potential drop V across the combination is the sum of potential drops V_1, V_2 and V_3 so -

$$V = V_1 + V_2 + V_3$$

$$\therefore \quad \frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

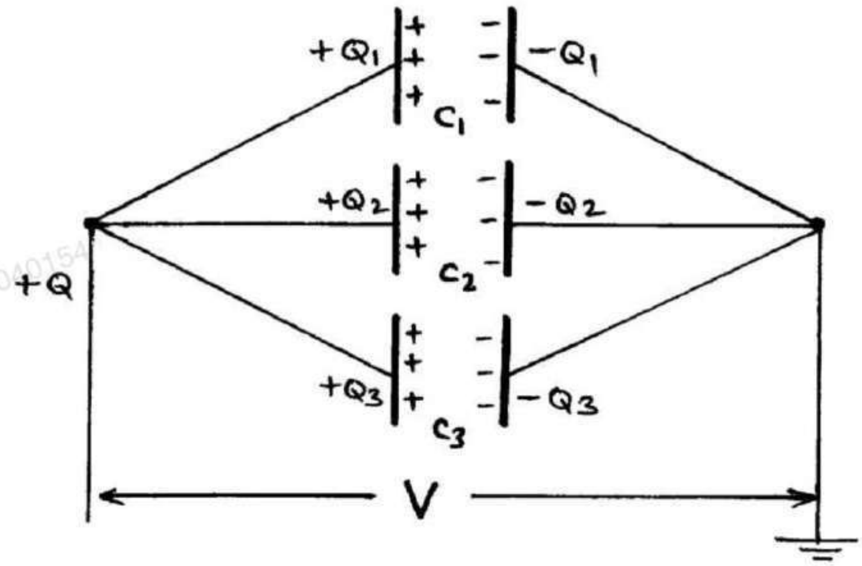
$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

for n capacitors connected in series, total capacitance would be -

$$\boxed{\frac{1}{C_s} = \sum_{i=1}^{n} \frac{1}{C_i}}$$

(ii) Capacitors in Parallel

Let three capacitors C_1, C_2 and C_3 are connected in parallel and V be the potential difference across the combination.



So the charges on the capacitors will be

$$Q_1 = C_1 V \quad , \quad Q_2 = C_2 V \quad \text{and} \quad Q_3 = C_3 V$$

So the total charge on the capacitors -

$$Q = Q_1 + Q_2 + Q_3$$

Let C_p be the equivalent capacitance in parallel

then $\quad C_p V = C_1 V + C_2 V_2 + C_3 V$

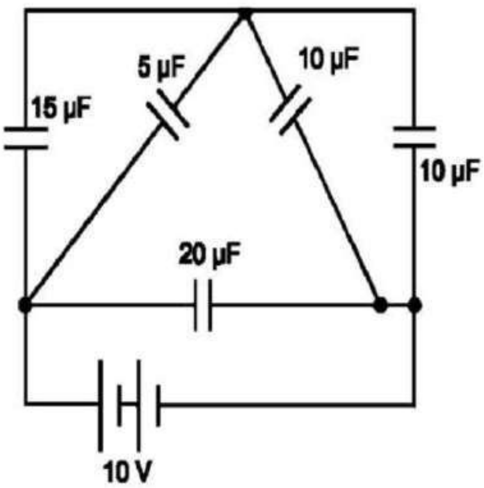
$$\boxed{C_p = C_1 + C_2 + C_3}$$

when n capacitors are connected in parallel then -

$$\boxed{C_p = \sum_{i=1}^{n} C_i}$$

Q. CBSE 2019

The figure shows a network of five capacitors connected to a 10V battery. Calculate the charge acquired by the 5 μF capacitor.



Sol. Net Capacitance of parallel C_1 and C_2 $= C_1 + C_2$

$$C_{12} = 15 + 5 = 20\,\mu F$$

Net capacitance of parallel C_4 and C_5 $= C_4 + C_5$

$$C_{45} = 10 + 10 = 20\,\mu F$$

C_{12}, C_{45} in series $\quad C_s = \dfrac{C_{12} \times C_{45}}{C_{12} + C_{45}} = \dfrac{20 \times 20}{20 + 20}$

$$C_s = 10\,\mu F$$

Capacitor C_3 in parallel with C_s, so that

$$C_p = 10 + 20 = 30\,\mu F$$

Potential difference across $C_s = 10V$

Potential difference across $C_{12} = C_{45} = 5V$

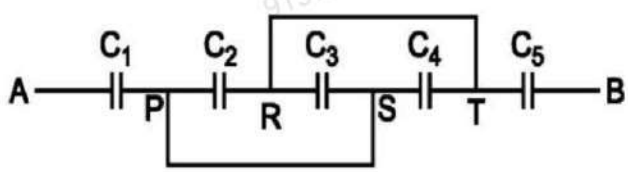
Charge on $5\,\mu F$, $Q = CV$

$$Q = 5 \times 10^{-6} \times 5$$
$$= 25 \times 10^{-6}\,C$$

Q. CBSE 2017

Find equivalent capacitance between A and B in the combination given below. Each capacitor is of $2\,\mu F$ capacitance.

Sol. Capacitors C_2 and C_3 and C_4 are in parallel



$$C_P = C_2 + C_3 + C_4$$

$$C_P = 2 + 2 + 2 = 6\,\mu F$$

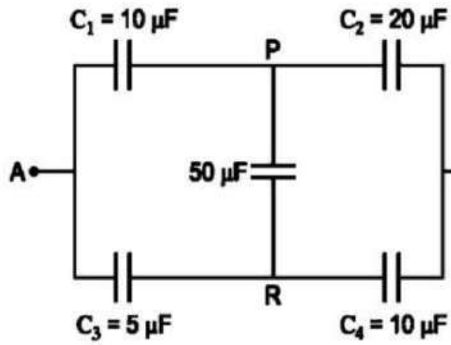
$\therefore$ Capacitors C_1, C_P and C_S are in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_P} + \frac{1}{C_S}$$

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{6} + \frac{1}{2} = \frac{7}{6}$$

$$C_{eq} = \frac{6}{7}\,\mu F$$

$\boxed{Q.}$ CBSE 2016 Calculate the equivalent capacitance between points A and B in the circuit below. If a battery of 10V is connected across A and B, calculate the charge drawn from the battery by the circuit.



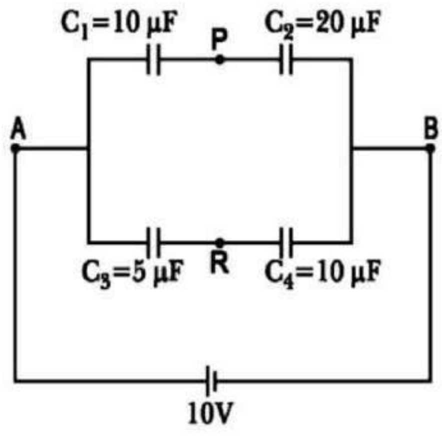
Sol. Here $\frac{C_1}{C_2} = \frac{C_3}{C_4}$

This is the condition of balanced wheatstone bridge, so there will be no current across 50 μF capacitor.

Now C_1 and C_2 are in series

$$C_{12} = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{10 \times 20}{10+20}$$

$$C_{12} = \frac{20}{3}\,\mu F$$



C_3 and C_4 are also in series

$$C_{34} = \frac{C_3 \times C_4}{C_3 + C_4} = \frac{5 \times 10}{5+10}$$

$$C_{34} = \frac{10}{3}\,\mu F$$

So that equivalent capacitance between A and B is

$$C_{AB} = C_{12} + C_{34}$$

$$C_{AB} = \frac{20}{3} + \frac{10}{3} = 10\,\mu F$$

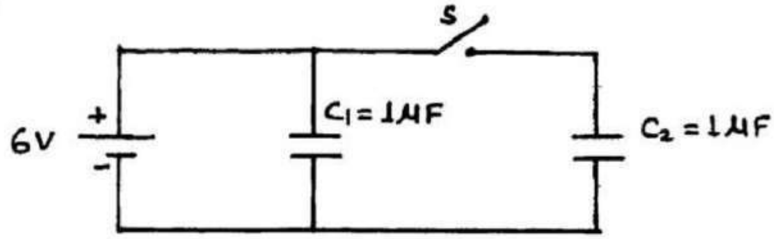
Hence, charge drawn from battery $Q = CV$

$$Q = 10 \times 10\,\mu C = 100\,\mu C$$

Q. Fig. shows two identical capacitors C_1, C_2 each of 1 μF capacitance connected to a battery of 6 V. Initially switch 's' is closed. After some time 's' is left open and dielectric slab of dielectric constant $K = 3$ are inserted to fill the space completely between the plates of the two capacitors. How will the-

CBSE 2010

(i) charge and (ii) Potential difference between the plates of the capacitors be affected after the slabs are inserted?



Sol. When switch s is closed both C_1 and C_2 are charged to 6V potential,

i.e. $V_1 = 6V$ and $V_2 = 6V$

When s is left open, C_1 is still connected to battery.

On introducing slab of dielectric constant, $K = 3$

$$C_1' = KC_1 = 3 \times 1 \times 10^{-6} = 3\,\mu F$$

$$V_1' = V_1 = 6\,V$$

$$Q_1' = C_1' V_1' = 3 \times 10^{-6} \times 6 = 18 \times 10^{-6}\,C$$

However, C_2 is disconnected from battery battery now, when s is left open. Due to introduction of slab,

$$C_2' = KC_2 = 3\,\mu F$$

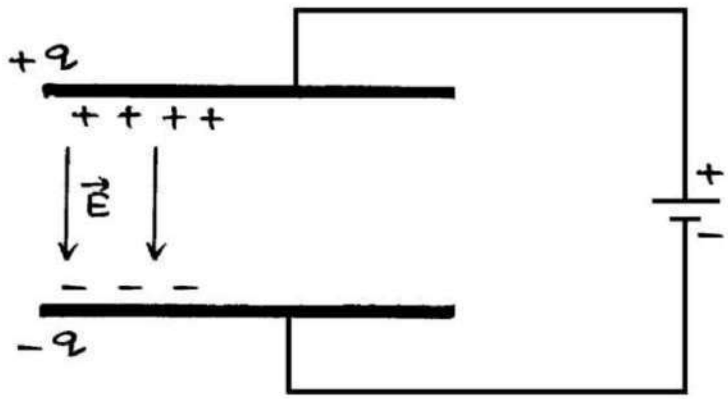
$$Q_2' = Q_2 = C_2 V_2 = 1 \times 10^{-6} \times 6 = 6 \times 10^{-6}\,C$$

$$\therefore \quad V_2' = \frac{Q_2'}{C_2'} = \frac{6 \times 10^{-6}}{3 \times 10^{-6}} = 2\,V$$

Energy Stored in Capacitor

The energy stored in a capacitor is the same as the work done to build up the charge on the plates by external source (battery) of voltage.

Let the two plates of the capacitor are initially uncharged. Now the charge is transferred to one plate in very small instalments of dq and equal opposite charge induces on the other plate.



At any intermediate stage, suppose charge on one plate is q and on other plate is −q.

∴ Potential difference between the plates $= \frac{q}{C}$

Small amount of work done in giving an additional charge dq to the capacitor is

$$dW = \frac{q}{C} \times dq$$

Total work done in giving a charge Q to the capacitor-

$$W = \int_{q=0}^{q=Q} \frac{q}{C}\, dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$\boxed{W = \frac{1}{2}\frac{Q^2}{C}}$$

This work is stored in the form of potential energy U.

$$U = W = \frac{1}{2}\frac{Q^2}{C}$$

as $Q = CV$ and $CV = Q$ so we can write-

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV}$$

Q. CBSE 2017 A 12 PF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor? If another capacitor of 6 PF is connected in series with it with the same battery, then find the charge stored and potential difference across each capacitor.

Sol. Electrostatic Energy stored $U = \frac{1}{2}CV^2$

$$U = \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2$$

$$U = 1.5 \times 10^{-8}\ J$$

Equivalent capacitance of 12 PF and 6 PF in series

$$C_s = \frac{C_1 \times C_2}{C_1 + C_2}$$

$$C_s = \frac{12 \times 6}{12 + 6} = 4\ PF$$

Charge stored across each capacitor

$$Q = CV$$

$$Q = 4 \times 10^{-12} \times 50 = 2 \times 10^{-10}\ C$$

* Is series combination charge on each capacitor is same.

$\therefore$ Potential difference across 12 PF capacitor

$$V_1 = \frac{2 \times 10^{-10}}{12 \times 10^{-12}} = \frac{50}{3}\ V$$

Potential difference across 6 μF capacitor

$$V_2 = \frac{2 \times 10^{-10}}{6 \times 10^{-12}} = \frac{100}{3}\ V$$

Q. (CBSE 2008) On charging a parallel plate capacitor to a potential V, the spacing between the plates is reduced to half and a dielectric medium of $\epsilon_r = 10$ is introduced between the plates, without disconnecting the DC source. Explain how the-
(i) Capacitance (ii) electric field and
(iii) energy density of the capacitor change?

Sol. Connected battery ensures same potential difference V across capacitor even after reducing the spacing between plates upto half and introducing the dielectric slab.

(i) If initial capacitance $C_1 = \epsilon_0 A/d$

Then final Capacitance $C_2 = \dfrac{10\epsilon_0 A}{d/2}$

$$C_2 = \frac{20\epsilon_0 A}{d} = 20C_1$$

Hence $C_2 = 20C_1$

Capacitance increases to 20 times of original value.

(ii) Initial electric field $E_1 = \dfrac{V}{d}$

Finally $E_2 = \dfrac{V}{d/2} = \dfrac{2V}{d} = 2E_1$

Hence $E_2 = 2E_1$

Electric field intensity gets doubled.

(iii) Energy density of the capacitor $U_1 = \frac{1}{2}\epsilon_0 E_1^2$

and $U_2 = \frac{1}{2}\epsilon_0 E_2^2$

$$U_2 = \frac{1}{2}\epsilon_0 (2E_1)^2 = \frac{1}{2}\epsilon_0 4E_1^2$$

so $U_2 = 4U_1$

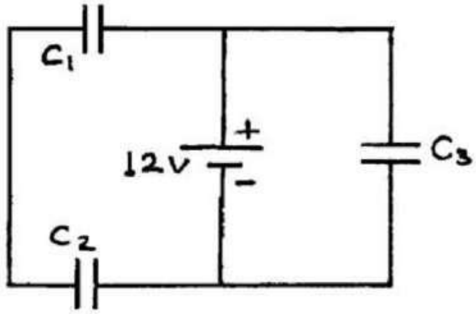
Hence energy density becomes 4 times of original value.

Q. Three identical capacitors C_1, C_2 and C_3 of capacitance 6 μF each are connected to a 12 V battery as shown-
CBSE 2009
Find

(i) Charge on each Capacitor

(ii) Equivalent Capacitance of the circuit

(iii) Energy stored in the network of capacitors.



Sol (i) Here $V = 12$ V

and $C_1 = C_2 = C_3 = 6\,\mu F = 6 \times 10^{-6}$ F

So the Charge on Capacitor C_3 is

$$q_3 = C_3 V = 6 \times 10^{-6} \times 12 = 72 \times 10^{-6}$$

$$q_3 = 72\,\mu C$$

since C_1 and C_3 are in series so the equivalent capacitance $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\frac{1}{C_s} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

$$C_s = 3\,\mu F$$

So the charge on each capacitor C_1 and C_2 is

$$q = C_s V = 3 \times 10^{-6} \times 12 = 36 \times 10^{-6} = 36\,\mu C$$

$\therefore$ Charge on each capacitor C_1 and C_2 is $36\,\mu C$.

(ii) Since C_1 and C_2 are in series whose equivalent Capacitance is $C_s = 3\,\mu F$

Now C_3 and C_s are in parallel so the equivalent Capacitance of the circuit is $C = C_3 + C_s$

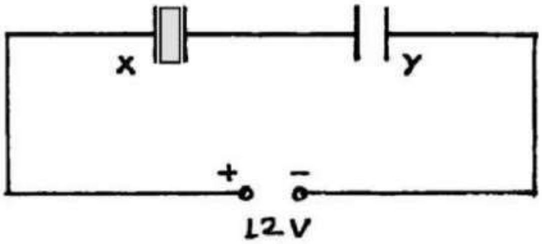
$$C = 6 + 3 = 9\,\mu F$$

(iii) Energy stored $U = \frac{1}{2} C V^2$

$$U = \frac{1}{2} \times 9 \times 10^{-6} \times (12)^2 = 6.48 \times 10^{-4}\ J$$

Q. CBSE 2009

Two parallel plate capacitors X and Y have the same area of plates and same separation between them, Y has air between the plates while X have a dielectric medium of $\epsilon_r = 4$.



(i) Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4 μF.

(ii) Calculate the potential difference between the plates of X and Y.

(iii) What is the ratio of electrostatic energy stored in X and Y ?

Sol. (i) Since capacitance C of the parallel plate capacitor is given by- $C = \frac{\epsilon_0 A}{d}$

$\therefore \quad Y = \frac{\epsilon_0 A}{d}$ and $X = \frac{\epsilon_0 K A}{d} = \frac{4\epsilon_0 A}{d}$

$\therefore \quad X = 4Y$

Since X and Y are in series combination

$\therefore \quad \frac{1}{4} = \frac{1}{X} + \frac{1}{Y} \Rightarrow \frac{1}{4} = \frac{1}{4Y} + \frac{1}{Y}$

or $\quad \frac{1}{4} = \frac{5}{4Y} \Rightarrow Y = 5\ \mu F$

So that $X = 4 \times 5 = 20\ \mu F$

(ii) Since $q = CV$

$q = 4 \times 10^{-6} \times 12 = 48 \times 10^{-6}\ C$

Potential diff. on capacitor X is given by

$V_x = \frac{q}{C_x} = \frac{48 \times 10^{-6}}{20 \times 10^{-6}} = 2.4\ V$

and potential diff. on capacitor Y is given by-

$$V_Y = \frac{q}{C_Y} = \frac{48\times10^{-6}}{5\times10^{-6}} = 9.6\ V$$

(iii) Energy stored in capacitor is $= \frac{1}{2}CV^2$

∴ Ratio of electrostatic energy stored in X and Y is

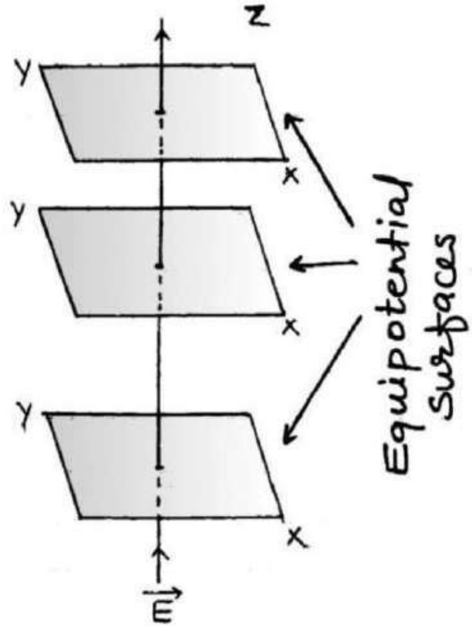
$$\frac{U_X}{U_Y} = \frac{\frac{1}{2}C_X V_X^2}{\frac{1}{2}C_Y V_Y^2} = \frac{20\times(2.4)^2}{5\times(9.6)^2}$$

$$\frac{U_X}{U_Y} = \frac{1}{4}$$

Q. Draw 3 equipotential surfaces corresponding to a field that uniformly increases in magnitude but remains constant along z-direction. How are these surfaces different from that of a constant electric field along z-direction?

CBSE 2009

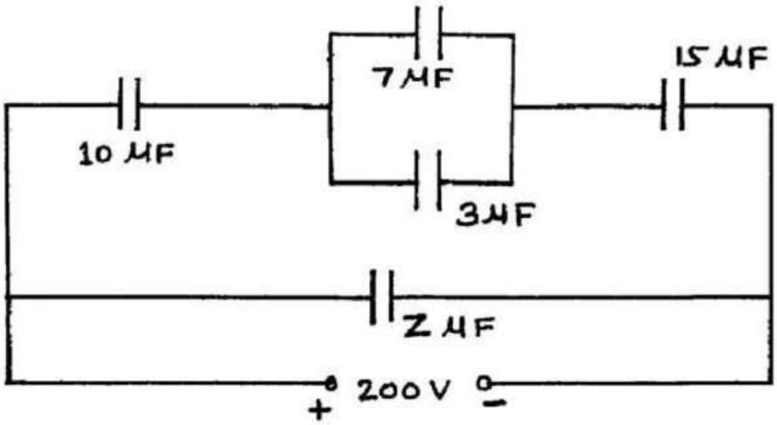
Sol. The three equipotential surfaces are in XY plane. As magnitude of field increases, distance between successive equipotential surfaces (for given dV) decreases as shown in figure.



If electric field were constant, distance between successive equipotential surfaces would stay constant.

Q. CBSE 2009

A system of capacitors connected as shown, has a total energy of 160 mJ stored in it. Obtain the value of the equivalent capacitance of this system and the value of Z.



Sol. Capacitor of 3 μF and 7 μF are in parallel.

$$\therefore \quad C_1 = 3 + 7 = 10 \ \mu F$$

Since 10 μF, 10 μF and 15 μF are in series -

$$\therefore \quad \frac{1}{C_2} = \frac{1}{10} + \frac{1}{10} + \frac{1}{15} = \frac{4}{15}$$

or $$C_2 = \frac{15}{4} \ \mu F$$

Hence total capacitance $C = C_2 + Z$

$$C = \left(\frac{15}{4} + Z\right) \mu F$$

$\therefore$ Energy stored $= 160$ mJ

$$\frac{1}{2} C V^2 = 160 \text{ mJ}$$

$$\frac{1}{2} \times \left(\frac{15}{4} + Z\right) \times 10^{-6} \times (200)^2 = 160 \times 10^{-3}$$

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$$\Rightarrow \quad \frac{15}{4} + Z = 8$$

$$Z = 8 - \frac{15}{4} = 4.25 \ \mu F$$

Q. (CBSE 2005) A parallel plate capacitor is to be designed with a voltage rating 1 KV using a material of dielectric constant 3 and dielectric strength about 10^7 V/m. For safety we would like the field never to exceed say 10% of the dipole strength. What minimum area of the plates is required to have a capacitance of 50 PF?

Sol. Maximum Permissible voltage $= 1\text{ KV} = 10^3$ V

Maximum Permissible electric field $= 10\%$ of 10^7 V/m

$= 10^6$ V/m.

Minimum separation required between the capacitor plates is -

$$d = \frac{V}{E} = \frac{10^3}{10^6} = 10^{-3}\text{ m}$$

So the minimum area required for the capacitor plates

$$A = \frac{Cd}{K\epsilon_0} = \frac{50\times10^{-12}\times10^{-3}}{3\times8.85\times10^{-12}}$$

$$A = 18.8\times10^{-4}\text{ m}^2 \approx 19\text{ cm}^2.$$

Q. (CBSE 2006) The electric field and electric potential at any point due to a point charge kept in air is 20 N/C and 10 J/C respectively. Compute the magnitude of this charge.

Sol. Electric field at distance r from the point charge $E = \frac{1}{4\pi\epsilon_0}\times\frac{q}{r^2} = 20$ N/C

Electric Potential at distance r from the point charge

$$V = \frac{1}{4\pi\epsilon_0}\times\frac{q}{r} = 10\text{ J/C}$$

so that $r = \frac{V}{E} = \frac{10}{20} = 0.5$ m

Hence Charge $q = 4\pi\epsilon_0 r\times V = \frac{0.5\times10}{9\times10^9}$

$q = 0.55\times10^{-9}$ C $\qquad [\because C = 4\pi\epsilon_0 r]$

Q.1. Determine the potential at a point 0.50 m (a) from a + 20 μC point charge (b) from a -20 μC point charge

Solution:

potential due to a point charge is $V = \frac{kQ}{r}$

(a) At a distance of 0.50 m from a positive 20 μC charge, the potential is

$$V = \frac{kQ}{r} = \left(9.0\times10^{9}\, N.m^{2}/C^{2}\right)\left(\frac{20\times10^{-6}C}{0.50m}\right)$$

$$= 3.6\times10^{5} V$$

(b) For the negative charge,

$$V = \left(9.0\times10^{9}\, N.m^{2}/C^{2}\right)\left(\frac{-20\times10^{-6}C}{0.50m}\right)$$

$$= -3.6\times10^{5} V$$

Q.2. Infinite charges of magnitude q are placed at coordinates x = 1m, 2m, 8m........ ∞ respectively along the x-axis. Find the value of potential at x = 0 due to these charges. $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$

Solution: Resultant potential at x=0

$$V = kq\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\cdots\cdots\right]$$

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$$= kq\left[\frac{1}{1-1/2}\right] = 2kq$$

Q.3. If 100J of work has to be done in moving an electric charge of 4C from a place where potential is -10V to another place where potential is V volt, find the value of V.

Solution:

Here; $W_{AB} = 100J$, $q_0 = 4C$,

$V_A = -10$ volt, $V_B = V$ volt

Since $V_B - V_A = \dfrac{W_{AB}}{q_0}$,

$$V - (-10V) = \frac{100J}{4C} = 25V$$

Or $V = 25V - 10V = 15V$

Q.4. A particle of mass m and positive charge q is released from point A. Its speed is found to be v when it passes through a point B. Which of the two points is at higher potential? What is the potential difference between the points?

Solution:

The point A is at higher potential than B as the particle has gained kinetic energy while moving from A to B.

The kinetic energy acquired by the particle is equal to loss in potential energy. i.e.,

$$\Delta K = q(V_A - V_B) \quad \Rightarrow \quad \frac{1}{2}mv^2 = q(V_A - V_B)$$

$$\Rightarrow \quad V_A - V_B = \frac{1}{2}\frac{mv^2}{q}$$

Q.5. *Calculate the work required to be done to make an arrangement of three particles each having a charge +q such that the particles lie at the vertices of an equilateral triangle of side a. What work will be done by electric field when the particles are shifted away so that the side of triangle becomes 2a?*

Solution:

The work required to make an arrangement of charges is equal to potential energy of the system

$$\therefore W = \frac{q^2}{4\pi \in_0 a} \times 3 = \frac{3q^2}{4\pi \in_0 a}$$

Work done by electric field represents the decrease in potential energy. So, we have

$$W_{E.Field} = -\Delta U = U_i - U_f$$

Now, $$U_i = \frac{q^2}{4\pi \in_0 (a)} \times 3$$

$$U_f = \frac{q^2}{4\pi \in_0 (2a)} \times 3$$

$$\therefore W_{E.Field} = \frac{3q^2}{4\pi \in_0 a} - \frac{3q^2}{8\pi \in_0 a} = \frac{3q^2}{8\pi \in_0 a}$$

Q.6. *A bullet of mass 2 gm is moving with a speed of 10 m/s. If the bullet has a charge of 2 micro coulomb, through what potential it be accelerated starting from rest, to acquire the same speed?*

Solution:

Use the relation $qV = \frac{1}{2}mv^2$;

Here, $m = 2 \times 10^{-3}\, kg$; $v = 10\, m/s$

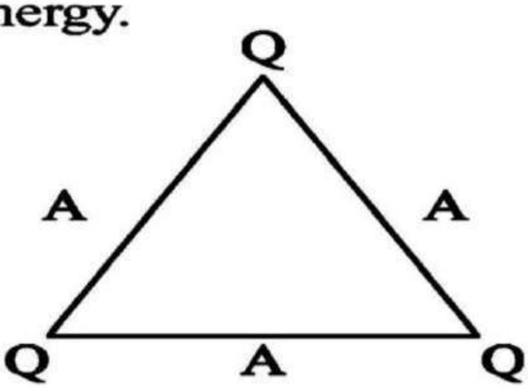
Therefore

$$V=\frac{mv^2}{2q}=\frac{2\times10^{-3}\times(10)^2}{2\times2\times10^{-6}}=\frac{10^{-1}}{2\times10^{-6}}$$

$$=5\times10^4\ volt=50\ kV$$

Q.7. Three equal charges Q are at the vertices of an equilateral triangle of side A. How much work is done (by an external agent) in bringing them closer to an equilateral triangle of side A/2?

Sol. In this problem first we have to calculate the initial and final potential energy and then the work done is final potential energy minus initial potential energy.



Initial potential energy $\frac{1}{4\pi\varepsilon_0}\frac{3Q^2}{A}$ (for each combination, the energy is $\frac{Q^2}{4\pi\varepsilon_0 A}$ and we have three such combinations)

$$\text{Final potential energy}=\frac{3Q^2}{4\pi\varepsilon_0 A/2}$$

$$\text{Now, work done } W=\frac{3Q^2}{4\pi\varepsilon_0 A/2}-\frac{3Q^2}{4\pi\varepsilon_0 A}$$

$$\text{Therefore, } W=\frac{3Q^2}{4\pi\varepsilon_0 A}$$

Q.8. If electric potential V at any point (x, y, z) all in meters in space is given by V = 4x² volt. Calculate the electric field at the point (1m, 0m, 2m)

Solution:

As electric field E is related to potential V through the relation

$$E = -\frac{dV}{dr}$$

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}(4x^2) = -8x = 8 \times 1 = 8$$

$$E_y = -\frac{dV}{dy} = -\frac{d}{dy}(4x^2) = 0$$

And $E_z = -\frac{dV}{dz} = -\frac{d}{dz}(4x^2) = 0$

So, $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k} = -8x\hat{i}$

at (1m, 0m, 2m) $\quad \vec{E} = -8\,\hat{i}$

i.e., it has magnitude 8 V/m and is directed along negative x-axis.

Q.9. An electric dipole in a uniform electric field E is turned from $\theta = 0$ position to $\theta = 60^0$ position. Find work done by the field.

Solution:

Work done by external agent

$(W) = pE(\cos\theta_1 - \cos\theta_2)$

(Where p is the dipole moment of the dipole).

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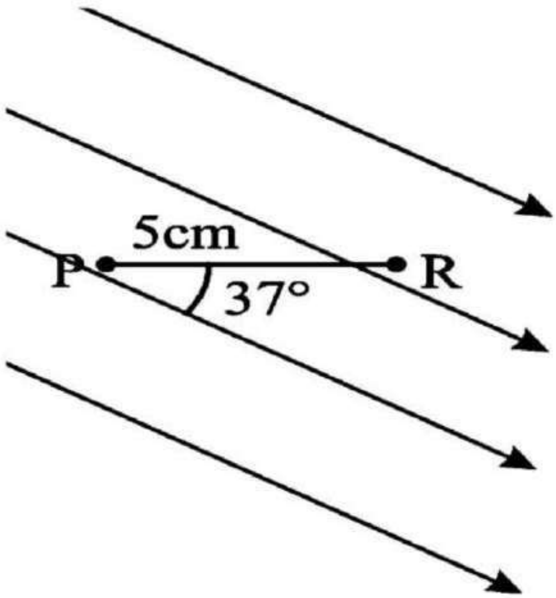
$$W = pE(\cos 0 - \cos 60) = \frac{1}{2}pE$$

Work done by the field $= -W = -\frac{1}{2}pE$

Q.10. A uniform field of magnitude 2000 N/C is directed 37⁰ below the horizontal as shown in the figure. Find (a) the potential difference between P and R. (b) If we define the reference level of potential so that potential at R is 500V, what is the potential at P ?



Solution:

(a) Here

$E = 2000 N/C, \theta = 37^0, V_R = 500V$

$dl = PR = 5cm = 5 \times 10^{-2} m$

$\vec{E}.\vec{dl} = Edl\, Cos\theta$

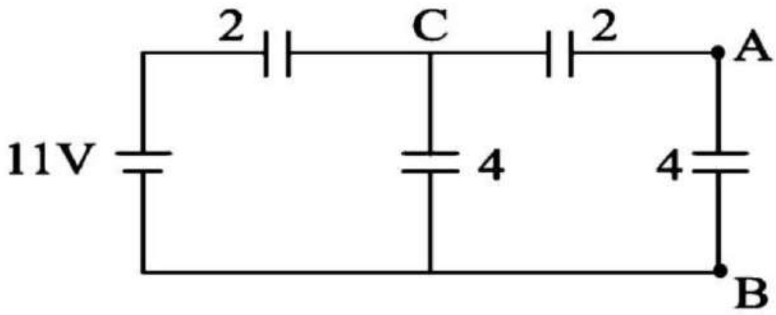
$= (2000 N/C)(5 \times 10^{-2} m) \cos 37^0 = 80V$

Thus, $V_P - V_R = \vec{E}.\vec{dl} = 80V$

(b) *As* $V_P - V_R = 80V$ and $V_R = 500V$,

$V_P = 500V + 80V = 580V$

Q.11. *Find the p.d. between the points A and B in the fig. The values of capacitances are in μF.*



Solution:

Capacitance of system

$$= \frac{2\times\left(4+\frac{2\times4}{2+4}\right)}{2+\left(4+\frac{2\times4}{2+4}\right)} = \frac{16}{11}\ \mu F$$

Charge on system

$$q = CV = 11 \times \frac{16}{11}\ \mu C = 16\ \mu C$$

Charge between A and B = Charge between

C and B $= q\cdot\frac{4/3}{4+4/3} = \frac{q}{4} = \frac{16}{4} = 4\ \mu C$

P.d. between A and B $= \dfrac{\text{Charge between A and B}}{\text{Capacity}}$

$$= \frac{4\ \mu C}{4\ \mu F} = 1\ \text{Volt}$$